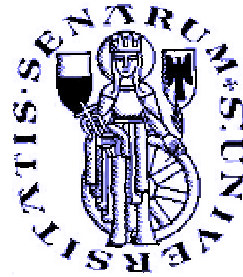


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# **Prioritarian Welfare Functions – An Elaboration and Justification**

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# Prioritarian Welfare Functions – An Elaboration and Justification

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**Abstract:** Apart from Rawls' maximin criterion, there are two main lines of correcting utilitarianism for considerations of distributive justice: egalitarianism (seeking to equalize utilities) and prioritarianism (giving more weight to improving the lot of those worse off). Though many people find prioritarianism appealing until now it has not been elaborated that much. The paper tries to help to fill several gaps left open. 1. A definition and mathematical distinction of egalitarian and prioritarian welfare functions will be given. 2. In an intuitive discussion of several candidates for prioritarian welfare functions one class of functions that are particularly apt to model prioritarian intuitions is filtered out, namely exponential functions. And some empirical findings are brought in for calibrating the functions' parameter for the degree of priority. 3. An internalistic justification of prioritarianism on the basis of sympathy is developed. Assuming an empirically founded (non-linear) function of our sympathy depending on the other person's well-being, it can be shown that prioritarianism optimises our sympathetic feelings.

## 1. Introduction: Advantages and Problems of Prioritarianism and the Aims of this Paper

Parfit [1995; 1997] has distinguished two types of correcting utilitarianism for considerations of justice, egalitarianism and prioritarianism. (Telic) egalitarianism seeks to diminish (or eliminate) interpersonal differences in personal goods, in particular individual utilities, as an intrinsic aim [Parfit 1997, 204]. Prioritarianism on the other hand wants each person to fare as well as possible, but is especially concerned with the worse off, i.e. "benefiting people matters more the worse off these people are" [Parfit 1997, 213; cf. Temkin 2003]. Whereas egalitarians are concerned with relativities, i.e. how each person's level compares with the level of other people, prioritarians are concerned with absolute levels, giving the higher priority to bettering the situation the lower the beneficiaries fare in absolute terms [Parfit 1997, 214].

Though egalitarianism for a long time, in particular to economists, seemed to be the right and only way of correcting utilitarianism for concerns about justice, since the 1970ies prioritarianism has been found attractive by many people, in particular philosophers (e.g. Frankfurt [1987], McKerlie [1984; 1994], Nagel [1978; 1991], Parfit [1995; 1997], Rabinowicz [2001; 2002], Raz [1986], and Wiggins [1987]) but economists as well, who at least used prioritarian welfare functions (e.g. Atkinson and Stiglitz [1980], Boadway and Bruce [1984], Drèze and Stern [1987], Fankhauser, Tol and Pearce [1997], Gaertner [1992; 1995], Sen [1973; 1984], Wagstaff [1991]). There are several intuitive reasons for this increasing popularity.

First, prioritarianism is a correction of utilitarianism with respect to equity or distributive justice; so it should be morally more adequate than utilitarianism.

Second, many people are *instrumentally* concerned with equality but do not see any *intrinsic* appeal of equality [Frankfurt 1997, further criticisms: Anderson 1999]: Why should it be an intrinsic moral or political ideal that if one person is well off (or badly off) all the other persons shall be equally well off? (Always comparing levels of well-being for criticizing deviations even seems to be a

nasty trait of character.) The sense of equity of a great part of these people requires that those who are badly off shall be supported first because their fate demands more for help. So prioritarianism for many people expresses the idea of equity in a more adequate way.

Third, another criterion not identical with prioritarianism but with prioritarian features, namely being primarily concerned with improving the lot of those worst off, is maximin or leximin [Rawls 1971; cf. Sen 1970, 138]. But leximin (in all relevant cases) disregards the lot of those better off, even of the persons second worst off, thus being hardhearted to them; and leximin is tremendously inefficient because it prefers even the tiniest advantage for the worst off instead of enormous advantages for those being better off. With respect to leximin and utilitarianism, prioritarianism is a synthesis, which preserves the advantages of both, namely efficiency and particular concern for those badly off, and avoids their disadvantages stemming from their respective one-sidedness. [Lumer 2000, 628-632; Temkin 2003]

Fourth, (basic) needs approaches are a further type of moral criterion with a prioritarian component and still nearer to prioritarianism than leximin: They give priority to the fulfilling of (basic) needs, i.e. they care about the fate of people up to a certain point, namely when their basic needs are fulfilled. These approaches are not hardhearted like leximin but they are *at least* unfriendly towards persons better off; and the point where moral engagement shall stop seems to be determinable only in an arbitrary manner. So prioritarianism seems to be morally more adequate than (basic) needs approaches, too.

In spite of these advantages prioritarianism until now has not been elaborated very much and among others the following problems still have to be resolved: 1. Egalitarianism as well as prioritarianism can be represented by concave welfare-functions. Does there then remain any difference between these two approaches and, if yes, what does it consist in? 2. If prioritarianism is to be applied in practice the exact kind of prioritarian (concave) welfare-function and the degree of priority has to be established. What exactly is the prioritarian welfare function? [McKerlie 2002.] 3. Prioritarians have described their intuitions about priority. And the justifications given so far are again only systematisations of moral intuitions. Is there any deeper, in particular internalist, i.e. motivational, justification for prioritarianism? - This paper tries to contribute to answering these three questions. The problem of justification (3) is probably the most fundamental one. The solution proposed in the following is to develop a justification of a prioritarian welfare function, based on our feelings of sympathy. Resolving the problem of justification should provide the fundamentals for developing a justified welfare function (2), too. Nonetheless I will proceed in the reverse order, i.e. first discuss in an intuitionist manner the right kind of prioritarian value function and only later examine for seeing only later on if the analytically developed and justified welfare function will be sustained by our intuitions too.

*Some terminology:* For convenience in the following text the result of a moral appraisal is called "value", abbreviated as "v", and the respective functions are abbreviated as "V". Prioritarian and egalitarian value functions are abbreviated as "VP" and "VE", respectively. The results of personal prudential appraisals are called "utilities" or "desirabilities" and abbreviated as "u", the

functions abbreviated as "*U*".<sup>1</sup> Sometimes value and desirability functions for certain prospects versus risky and uncertain prospects have to be distinguished. In this case the abbreviations of functions for certain prospects are extended by a "*T*" (for "total") and those for not certain prospects with a "*P*" (for "prospect (in the narrow sense)"), leading e.g. to the abbreviations "*UT*" for "total utility" or "*VPP*" for "prioritarian prospect value". Individual utilities and their moral values will sometimes be normalized here in the interval [0, 1]. In this case the utility will be that of a complete life with utility 0 referring to a life equivalent to not having lived at all and 1 referring to such an extremely good life that it is rarely attained in a society; negative utilities, though being conceivable, empirically will not exist because for people in danger of approaching that level it is not only rational and psychologically hard to avoid committing suicide but in the end they would die of grief anyway. But the following considerations do not rely on this psychological hypothesis. The normalization introduced may be understood as a convenient restriction of representation only, which does not exclude that in very rare cases there are utilities and moral values of individual utilities outside the interval [0, 1].

## 2. Characteristic Features of Prioritarianism and Egalitarianism

### 2.1. Defining 'Prioritarianism'

*Utilitarianism* in equating moral value with the sum (or the mean) of all individual utilities treats all improvements equally, i.e. gives them the same weight - independent of how well off the persons are to whom the improvements go. Utilitarianism in this respect does not care for distributive justice. For correcting this Rawls [1971, 302 f.] proposed the *maximin* criterion (which says that from a set of distributions the one that entails the highest level for the worst off is morally best), or more precisely: the *leximin* criterion [1971, 83] (which adds to the maximin criterion that if in two distributions the worst off fare equally bad the one that gives more to the second worst off is better, and that if even the second worst off fare equally bad the level of the third worst off counts etc.).<sup>2</sup> The historical starting point for prioritarianism then was that some people

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<sup>1</sup> In other contexts I distinguish sharply between desirability and utility, reserving "desirability" to results of appraisals satisfying much stronger prudential requirements [cf. Lumer 2000, 241-548]. But in the present context this is of little importance.

<sup>2</sup> Some qualifications: Rawls does not apply the maximin principle to utilities but to income and social positions. However in the first edition of "A Theory of Justice" he added a description of the "general conception", saying that the maximin principle should be applied to all primary goods: liberty, opportunity, income, wealth, the bases of self respect [Rawls 1971, 303], which may invite readers to further radicalize and extend this idea to utilities. This "general conception" has been deleted in the second edition, though [cf. Rawls 1979, 337; Rawls <sup>2</sup>1999, 267]. - Leximin has been invented by Sen [1970, 138] for correcting the obvious insufficiency of maximin. And Rawls quotes Sen in this respect [Rawls 1971, 83]. But Sen did not adopt leximin (at least not explicitly), and Rawls immediately after introducing leximin returned to maximin

(philosophers and economists alike) were fascinated by Rawls' idea of correcting utilitarianism in giving priority to the improvement of the situation of those worse off but that they rejected the absolutistic feature of giving *infinite* priority to those worst off. To prefer a minor improvement to a bigger improvement because the minor improvement goes to a person worse off is plausible - but only to a certain degree. There must be some point where the improvement for the one better off is so much bigger than the improvement for the worse off or where the number of the better off who would profit from the measure is so high that eventually the moral preference should reverse in favour of the measure to the advantage of the better off [Nagel 1978; Nagel 1991, ch. 7.; Sen 1984]. Parfit has coined the term "*priority view*" for this kind of reasoning and defines it this way: "Benefiting people matters more the worse off these people are" [Parfit 1997, 213]. (This definition has been quoted very often [cf. e.g. Temkin 2003; Fleurbaey 2002, 4].) Though Parfit's definition uses comparatives prioritarianism is not primarily interested in comparing peoples' well-being but in their absolute levels of well-being. Benefiting the worse off matters more not because they are worse off than others but because they are at a lower absolute level [Parfit 1997, 214]. A somewhat different way of explicating prioritarian intuitions is to take prioritarianism as a synthesis of utilitarianism and leximin somewhere between these two systems, which preserves the advantages of both, utilitarianism's efficiency and leximin's concern for those badly off, and removes their respective disadvantages, utilitarianism's neglect of distributional justice and leximin's inefficient and hard-hearted intrinsic disregard of improvements for those better off (even the second worst off) [Lumer 1997, 102; Lumer 2000, 628-632; Temkin 2003].

'Prioritarianism' then may be defined informally as follows:

*Prioritarianism* is a way of intrinsically morally valuing according to which all changes in personal desirabilities are valued in strict positive correlation to these changes but giving more - though not infinitely more - importance or weight to changes for people being badly off; this importance declines continuously and smoothly with increasing personal desirability levels of the persons looked at, however without ever reaching zero - not even for the highest levels.

The different weights express the degree of our moral concern, i.e. how much improving the lot of the person in question is near to the moral subject's heart. Because this desirability function is applied to life situations of *individuals* the moral value of a community's or society's position can be established additively - which presupposes interpersonal comparability of personal desirabilities.

The straightforward way of formally modelling prioritarian valuing is to define a one-adic moral value function  $VP$  over normalized personal desirabilities and to define the moral value of some option  $a$  as the sum of the moral values of all the individuals' ( $i$ ) desirabilities of  $a$ :

$$VPT(a) := \sum_i VP(UT_i(a)) = \sum_i VP(u_i).$$

The prioritarian moral value function  $VP$  increases monotonously because of the strict positive correlation between personal desirability changes and their moral value (cf. figure 1). (Because of the normalization it has to cross the points  $\langle 0, 0 \rangle$  and  $\langle 1, 1 \rangle$ .) For convenience the function should

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[Rawls 1971, 83] because, as he later wrote, he thought that in practice there would not be any difference between the two principles [Rawls 1979, 103 f.; Rawls 1999, 72 f.].

be three times differentiable. Monotonous increase then is equivalent to the first derivation being above 0 throughout ( $VP' > 0$ ). The slope or the first derivation  $VP'$  of the value function expresses the moral weight attributed to personal desirability changes occurring to people with a given desirability level, or the degree of concern for people at that level. (The steeper the slope, i.e. the higher the first derivation, the more moral value is attributed to an incremental increase in utility of a person with the respective utility level. That the life of a person with an already high utility level is morally highly valued too (high  $VP$ ) is correct because otherwise e.g. bringing people at that level would not be a morally very good thing. But further improvements are valued comparatively little, though always positively.) The informal definition of 'prioritarianism' given above as well as Parfit's definition mainly speak of the moral weight attributed to changes, i.e. they speak of the first derivation  $VP'$  of the value function. They say that this weight, i.e. the first derivation, declines continuously and smoothly but always remaining above 0 [cf. Rabinowicz 2001]. That it *declines* continuously means that the second derivation  $VP''$  is negative throughout ( $VP'' < 0$ ). And the *continuous and smooth* decline that does not lead to 0, though perhaps approaching 0 for high values of  $u$ , means that this first derivation  $VP'$  is strictly convex.<sup>3,4</sup> This holds because a *smooth* decline implies that the derivation  $VP'$  may not decline stronger at some point, thus perhaps turning to concavity for some while, and later on more slowly, thus returning to convexity. (Turning to concavity at some point would mean that prioritarianism would punish in a certain way exceeding

<sup>3</sup> A one-adic function  $F$  is *strictly convex* in the interval  $[a, b]$  iff: for all  $x_1, x_2 \in [a, b]$  and for all  $h$   $0 < h < 1$  holds:  $F(h \cdot x_1 + (1-h) \cdot x_2) < h \cdot F(x_1) + (1-h) \cdot F(x_2)$ . Strict convexity means that for all pairs of  $x_1, x_2$  (from the interval  $[a, b]$ ) the curve between  $F(x_1)$  and  $F(x_2)$  lies below the cord directly connecting  $F(x_1)$  and  $F(x_2)$ . The curve is "bulged" towards the bottom so to speak. Strict convexity for twice differentiable functions is equivalent to  $F''(x) > 0$  (in the interval  $[a, b]$ , of course). *Quasi-convexity* softens the conditions given above in substituting the "<" of the definition by " $\leq$ ", thus allowing for straight pieces of the curve where the curve does not lie below the cord but on the cord. Quasi-convexity for twice differentiable functions is equivalent to  $F''(x) \geq 0$ . *Concavity* is the opposite to convexity and means that a curve is "bulged" upwards. I.e. for all pairs of  $x_1, x_2$  from the interval  $[a, b]$  the curve between  $F(x_1)$  and  $F(x_2)$  lies above (for strict concavity) or above or on (for quasi-concavity) the cord directly connecting  $F(x_1)$  and  $F(x_2)$ . In the above definition the "<" has to be substituted by ">" (for strict concavity) and " $\geq$ " (for quasi-concavity), respectively. For twice differentiable functions strict concavity is equivalent to  $F''(x) < 0$ , and quasi-concavity is equivalent to  $F''(x) \leq 0$ .

A more general definition, which holds for  $n$ -adic functions too, runs as follows. A social welfare function  $V$  is *strictly convex* iff: let  $u^{\circ}_1$  and  $u^{\circ}_2$  be two  $n$ -tuples of utilities, then for all  $h$   $0 < h < 1$  holds:  $V(h \cdot u^{\circ}_1 + (1-h) \cdot u^{\circ}_2) < h \cdot V(u^{\circ}_1) + (1-h) \cdot V(u^{\circ}_2)$ , where  $h \cdot u^{\circ}_1$  means that all elements of  $u^{\circ}_1$  have to be multiplied by  $h$  [cf. Sen 1973, 20].

<sup>4</sup> Fleurbaey interprets Parfit's definition "benefiting people matters more the worse off these people are" this way: "At first glance, this seems to be just saying that individual weights should be (positive and) inversely related to the individual initial levels of benefits" [Fleurbaey 2002, 4]. "Inversely related", strictly speaking, would be identical to  $1/x$  being the first derivation  $VP'$  (which expresses the weight attributed to further improvements); and this implies that the value function  $VP$  itself would be  $\ln(x)$  (because  $1/x$  is the first derivation of  $\ln(x)$ ).  $\ln(x)$ , being monotonously increasing and strictly concave itself and having the strictly concave first derivation  $1/x$ , is a possible candidate for prioritarian value functions. But it is a rather particular one, and at the present stage of consideration other particular value functions fulfilling the other conditions should not be excluded. Probably Fleurbaey, too, did not mean "inversely related" in the strict sense so that the more general interpretation given here, namely positive but monotonously falling and strictly convex, would specify his understanding too.

certain arbitrary threshold desirabilities; but prioritarianism is neither arbitrary in this way nor hostile to high utilities.) And approaching to 0 without reaching 0 even for very high  $u$  (together with the exclusion of infinite weights of  $VP'$  for the smallest utilities) excludes linearity (because a linear and decreasing  $VP'$  would somewhere intersect the  $x$ -axis), and thus excludes mere quasi-convexity of  $VP'$ . Strict convexity of  $VP'$  is equivalent to the second derivation of this function ( $VP''$ ) being above 0 throughout; and because the second derivation of  $VP'$  is identical to the third derivation  $VP'''$  of the value function  $VP'$  itself this means:  $VP''' > 0$  throughout.

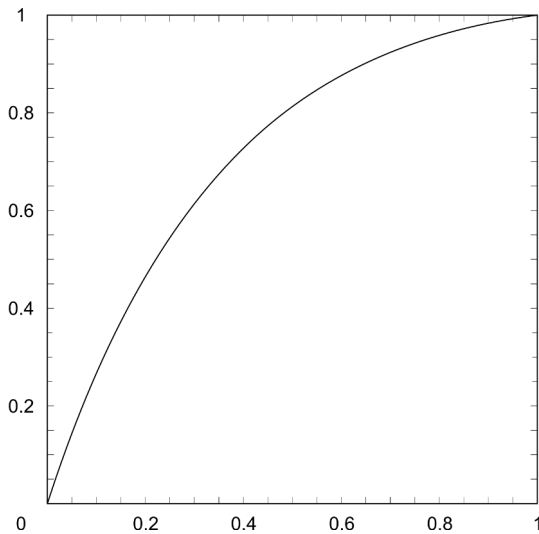
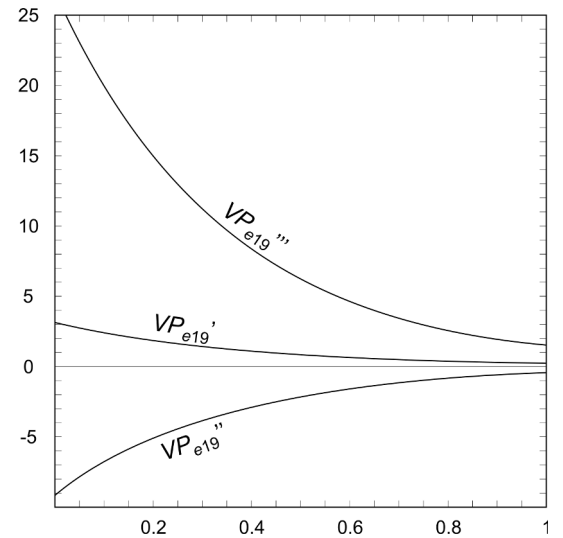
Fig. 1a: Prioritarian value function  $VP$  ( $VP_{e19}$ )

Fig. 1b: Derivations of prioritarian value function



So we may define 'prioritarianism' formally this way:

*Prioritarianism* is a way of moral valuation that is representable by an

(P1) additively separable moral value function of the form:

$$VPT(a) := \sum_i VP(UT_i(a)) = \sum_i VP(u_i) \text{ for certain prospects } a \text{ and}$$

(P2)  $VPP(a) := R[\langle VPT(a_1), P(a_1) \rangle, \dots, \langle VPT(a_m), P(a_m) \rangle]$  for risky and uncertain prospects  $a = (\langle a_1, P(a_1) \rangle, \dots, \langle a_m, P(a_m) \rangle)$  –  $a_i$  is a possible outcome of  $a$ , and  $P(a_i)$  is its probability –, where

(P3)  $R(x_1, \dots, x_m)$  is a suitable monotonously increasing weighting function for not certain prospects with  $R(0) = 0$  and  $R(\langle VPT(a), 1 \rangle) = VPT(a)$ , and where

(P4)  $VP(u)$  is a three times differentiable value function with

(P4.1)  $VP'(u) > 0$  for all  $u$ ,

(P4.2)  $VP''(u) < 0$  for all  $u$ ,

(P4.3)  $VP'''(u) > 0$  for all  $u$ , and

(P5) for which a set of real (at some point in history) options  $\{a, b\}$  exists with  $VPP(a) > VPP(b)$  in contrast to a leximin valuation (because  $a$  entails some bigger utility for people better off than  $b$  for some people worse off).

Conditions P2 and P3 deal with risky and uncertain prospects. The present paper does not intend to tackle the problems of an ethics of risk. So I have essentially left open the form of the

weighting function  $R$  for risky and uncertain prospects. It may be the expected moral value of the prospect (so that  $R(\langle v_1, p_1 \rangle, \dots, \langle v_m, p_m \rangle) = \sum_{j=1}^m v_j p_j$ ) or some non-linear weighting function. In the following I will mainly disregard these questions, dealing with  $VPT$  only. But one determination is important even in the present perspective. There are two main ways of morally locating risk.<sup>5</sup> Risk may be located in the individuals' utilities so that first the individual prospect desirabilities ( $UP_i(a)$ ) of the respective risky option  $a$  have to be calculated, to which subsequently the prioritarian value function has to be applied in the fashion of P1. (P1 would then change to:  $VPP(a) := \sum_i VP(UP_i(a))$ , and P2 and P3 could be deleted.) Or risk may be located in the social or moral value, so that first the moral value of the possible outcomes has to be established, to which subsequently the risk weighting procedure has to be applied. This is the line followed in the above definition of 'prioritarianism'. Because  $VP$  is a non-linear function these two ways of locating risk and calculating moral value often lead to different results.<sup>6</sup> So in order to avoid inconsistencies one of these two ways has to be fixed. Here the social or moral localization of risk has been chosen because it leads to more adequate results.<sup>7</sup> First, prioritarianism is a valuation from a moral point of view. So risk should be valued from this point of view, too. Risk then means that some total *moral value* will come about with a certain probability only. And we should first calculate the value of what morals really is interested in (i.e. the total moral value of the outcomes) and only subsequently deal with the risk. In addition, moral and individual handling of risk may differ, e.g. some individual may be risk seeking whereas the moral perspective could be risk averse. For giving the primate to the moral perspective social risk localization has to be chosen. Second, individual localization of risk implies some sort of prioritarianism over (individual) opportunities. People may have had good or medium opportunities but end up in misery. What a prioritarian should be concerned about is the latter fact, that those people end up in misery, and not about the mere opportunities. A biting criticism directed against equality of opportunities says: Equality of opportunity "instead of reducing the huge gap between, say, physicians and ditch diggers, it might merely change the demographic composition of those groups" [Temkin 1993, 85 fn]. Something similar holds for prioritarianism over opportunities: Instead of improving with some priority the lot of ditch diggers it might lead to fictitious improvements by introducing lotteries over both social positions that result in essentially the same distributions but with different demographic

<sup>5</sup> One might even try to mesh the two ways of calculation so that there would be a third, impure way of calculation [cf. Fleurbaey 2002, 11 f. with further references]. But given the strong arguments against individual risk localization (cf. below) and the fact that Fleurbaey's argument for such meshing has to do with egalitarian considerations, this kind of meshing can be disregarded here.

<sup>6</sup> Think of a risky prospect  $a$  affecting one person  $i$  only, leading with a probability of 0.5 to an outcome with a total utility of 1 and with a probability of 0.5 to an outcome with the total utility 0 ( $a = (\langle 1, 0.5 \rangle, \langle 0, 0.5 \rangle)$ ). If we take expected utility and expected moral value respectively to be the best ways of dealing with risk and  $VP(0.5)=0.813$  (remember that  $VP$  is a concave function leading from  $\langle 0, 0 \rangle$  to  $\langle 1, 1 \rangle$  so that  $VP(0.5)$  has to be higher than 0.5) we get the following moral values. For individual risk location  $VPP(a) = VP(UP_i(a)) = VP(1 \cdot 0.5 + 0 \cdot 0.5) = VP(0.5) = 0.813$ . For social risk location  $VPP(a) = R(\langle VP(UT_i(a_1)), 0.5 \rangle, \langle VP(UT_i(a_2)), 0.5 \rangle) = R(\langle VP(1), 0.5 \rangle, \langle VP(0), 0.5 \rangle) = R(\langle 1, 0.5 \rangle, \langle 0, 0.5 \rangle) = 0.5$ . So the individual risk location leads to a higher moral value of  $a$  than the social risk location.

<sup>7</sup> Rabinowicz [2001] goes the same way.



compositions. (This holds because the moral value of a lottery assigning to one person the position of a ditch digger to the other that of a physician would be enormously higher than the moral value of assigning these positions with certainty.<sup>8</sup>) And this seems to be absurd. - A different strategy of defining 'prioritarianism' leading to a broader definition would have been this. In the definition the question of dealing with not certain prospects is left open and it is required only that (for avoiding inconsistencies) any prioritarian welfare function has to choose and keep fix one of the two ways exposed. But first, all prioritarians dealing with the problem in question as far as I know tend towards social localization of risk because they are not primarily interested in giving priority to improving worse opportunities but in priority to improving worse levels of well-being. And second, fixing one way at this point facilitates the following presentation.

Condition P5 is necessary for distinguishing prioritarianism from leximin because the value function  $VP$  otherwise could be so strongly convex, i.e. approaching to the right angle  $\langle 0, 0 \rangle$ ,  $\langle 0, 1 \rangle$ ,  $\langle 1, 1 \rangle$ , that it would rank all options exactly like leximin. Condition P5 is very weak in requiring only one ranking of real options in opposition to leximin. P5 thus still allows for a very wide range of prioritarian welfare functions, including among others very radical forms of prioritarianism.

Some implications of the just developed definition of 'prioritarianism' are.

1. The moral value function  $VPT$  is universalistic (or symmetrical as economists say) with respect to the beneficiaries, i.e. it applies the same weighting function  $VP$  to the utilities of all persons.
2. Because of P4.1 (and P1) prioritarianism fulfils the Pareto principle for certain prospects, though because of the social localization of risk (i.e. to apply moral valuations to total utilities of outcomes and not to expected utilities, cf. P3) it does not fulfil the Pareto principle for risky and uncertain prospects throughout.<sup>9</sup> I do not think that this is a big disadvantage because fulfilling the Pareto principle for certain prospects already reflects the spirit of the Pareto principle.

<sup>8</sup> This may be illustrated by using the figures used in footnote 6. The certain prospect is:  $a = \langle 1, 0 \rangle$ , which means that  $a$  implies the utility distribution  $\langle 1, 0 \rangle$  for sure. The lottery may be:  $b = (\langle 1, 0 \rangle, 0.5), \langle 0, 1 \rangle, 0.5)$ , which means that with equal chances it leads to the distribution  $\langle 1, 0 \rangle$  and  $\langle 0, 1 \rangle$ , respectively. Now, according to individual risk localization:  $VPP(a) = VP(UE_1(a)) + VP(UE_2(a)) = VP(1) + VP(0) = 1+0 = 1$ ;  $VPP(b) = VP(UE_1(b)) + VP(UE_2(b)) = VP(0.5) + VP(0.5) = 0.813+0.813 = 1.626$ . So  $b$  would be counted as a moral improvement compared to  $a$ . According to social or moral risk localization  $a$  and  $b$  are of equal value:  $VPP(a) = R(\langle VPT(a), 1 \rangle) = R(\langle (VP(UT_1(a)) + VP(UT_2(a))), 1 \rangle) = R(\langle (VP(1)+VP(0)), 1 \rangle) = R(1, 1) = 1$ ;  $VPP(b) = R(\langle VPT(b_1), 0.5 \rangle, \langle VPT(b_2), 0.5 \rangle) = R(\langle VPT\langle 1, 0 \rangle, 0.5 \rangle, \langle VPT\langle 0, 1 \rangle, 0.5 \rangle) = R(\langle (VP(1)+VP(0)), 0.5 \rangle, \langle (VP(0)+VP(1)), 0.5 \rangle) = R(\langle (1+0), 0.5 \rangle, \langle (0+1), 0.5 \rangle) = R(\langle 1, 0.5 \rangle, \langle 1, 0.5 \rangle) = 1$ .

<sup>9</sup> An example for a violation of the Pareto principle for risky prospects would be this: If to a person having an utility level of 0.5 for sure ( $a = \langle 0.5, 1 \rangle$ ) is offered a lottery  $b$  with equal chances of ending up with 1 or 0.1 ( $b = (\langle 1, 0.5 \rangle, \langle 0.1, 0.5 \rangle)$ ) and that does not affect anybody else, changing from  $a$  to  $b$  would be a Pareto improvement because  $b$  has a higher expected utility ( $UE_i(b) = 1 \cdot 0.5 + 0.1 \cdot 0.5 = 0.55 > 0.5 = UE_i(a)$ ). The moral prioritarian ranking mostly will be inverted though, e.g. attributing the following values:  $VPP(a) = VP(0.5) = 0.813 > VPP(b) = R(\langle VPT(b_1), 0.5 \rangle, \langle VPT(b_2), 0.5 \rangle) = R(\langle VP(1), 0.5 \rangle, \langle VP(0.1), 0.5 \rangle) = R(\langle 1, 0.5 \rangle, \langle 0.269, 0.5 \rangle) = 0.635$ . - From the individual's standpoint the moral valuation seems to be strongly risk averse, whereas from the moral standpoint it is risk neutral and a consequence of subjecting individual utilities to a non-linear moral valuation.

3. Prioritarianism fulfils the Pigou-Dalton condition applied to well-being (measured in utilities). This condition says: Every transfer of a fix amount of utility  $d$  from a person  $i$  with a higher utility level  $u_i$  to a person  $j$  with a lower utility level  $u_j$ , making  $i$  not worse off than  $j$  shall be valued as a moral improvement, i.e. if  $u_i - d > u_j + d$  the transfer of  $d$  should count as an improvement [Pigou 1912, 24; Dalton 1920, 351; Sen 1973, 27]. (A stronger version says that the redistribution shall not make  $i$  as bad off as  $j$  formerly so that the restriction is:  $u_i - d > u_j$  [Atkinson 1970, 247].) Fulfilment of the Pigou-Dalton condition is implied by the strict concavity of the value function  $VP$  (i.e. P4.2). (Increase of the value function (i.e. P4.1) is not necessary for fulfilling the Pigou-Dalton condition.)
4. The requirement that the prioritarian value function is "additively separable" means that it is a function of the form:  $F(u_1) + F(u_2) + \dots + F(u_n)$ . And this means that the well-being of every person is valued separately and absolutely, independent of its relation to the well-being of others [Broome 2002, 1 f.]. This implies that moral valuation can be applied even to the situation of single individuals. (A case in question may be decisions about supererogatory acts in which the subject reflects if such action will create enough moral value to merit being preferred to furthering one's own interests. Remember that supererogatory acts do not require moral optimising and hence no comparison of the benefit of this action to the possible benefit of its alternatives to other persons. (McKerlie [2002, 10-12] is looking for similar examples.)) And it implies that the moral valuation of the well-being of subgroups is independent of the well-being of the rest of the group; so if the rest of the group is not affected by the options under decision its well-being can be disregarded [Klint Jensen 2002, 18 f.].<sup>10</sup>
5. Because prioritarianism values absolute levels of well-being with non-linear value functions it seems to require utility measurements on a ratio scale level (i.e. a measurement with a fixed zero point). Even though this actually is not true, prioritarian value functions have to be chosen with respect to a fixed utility measure with interpersonally comparable utility points. And before applying the value function to further person's utilities these utilities first have to be transformed into that standardized utility measure via positive-linear transformation. If prioritarian valuation shall be applied to utilities of different utility measures (resulting from the standardized utilities by positive-linear transformation) the value function  $VP$  has to be subjected to a compensatory adjustment [Rabinowicz 2001, n. 10].

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<sup>10</sup> Fleurbaey objects to including additive separability in the definiens of 'prioritarianism'. Leximin shall be a limiting case of prioritarianism, but leximin is clearly comparative - is speaks e.g. of the "worst off", i.e. those being worse off than everybody else - and therefore does not allow additively separated valuations [Fleurbaey 2002, 5]. Indeed, the *definiens* of 'leximin' speaks of comparisons, and therefore, leximin cannot be *intensionally* identical to extreme prioritarianism. However the two welfare functions can be made *extensionally* equivalent by choosing such an extremely concave prioritarian value function  $VP$  that its application in all cases leads to the same ranking as leximin. Therefore P5 had to be introduced in the definition of 'prioritarianism' for making prioritarianism even extensionally different from leximin.

## 2.2. Defining 'Egalitarianism'

In the present subsection 'egalitarianism' will be defined in a rather narrow and specific sense as it is used by many philosophers and which is in opposition to 'prioritarianism', namely as a way of moral valuation striving for equality. Economists today use "egalitarianism" in a much, much broader sense including both, prioritarianism and egalitarianism in the narrow sense, as well as systems that are neither prioritarian nor egalitarian. The specific welfare economist version of this *egalitarianism in the broad sense* is roughly identical to a welfarist way of moral valuation (i.e. a valuation (mainly) consisting in an aggregation of individual utilities) that is different from utilitarianism in trying to include considerations of distributive justice. Most of these egalitarian valuations will satisfy the Pigou-Dalton condition. But because this is not the focus of this paper I would like to leave it open if the Pigou-Dalton condition should be included in a respective definition. Economists usually (and some philosophers too) are reluctant to give up their broad usage of "egalitarianism" [cf. e.g. Fleurbaey 2002; Klint Jensen 2002, 3; Hausman 2001, 1]. One simple terminological reason for this is that in the present discussion there is no other common term that could fill the gap if "egalitarianism" were used in a narrower sense, so that the whole project of trying to correct utilitarianism by considerations of distributive justice would remain without name. Therefore I propose to coin a new term and to call "*welfare equitism*" what welfare economists so far have called "egalitarianism" (in the broad sense), namely: a welfarist way of moral valuation that includes aspects of distributive justice in its rule of aggregating utilities. With such a new term at hand we are free to use "egalitarianism" in a more appropriate way, i.e. the narrow sense.

Parfit has distinguished *telic egalitarianism* from *deontic egalitarianism*, where the former is interested in the final distribution and the latter in a certain way of acting. Intrinsic egalitarianism has to be distinguished from instrumental egalitarianism: For *intrinsic egalitarianism* the intrinsic value is defined directly dependent on some distributive pattern, whereas *instrumental egalitarianism* is interested in certain distributions because they increase the realization of some other value.<sup>11</sup> In addition, egalitarianisms have to be distinguished according to the good they hold to be distributed equally: well-being, income, power etc. [Cf. Parfit 1997, 203-209.] In the following I will speak only of telic and intrinsic egalitarianism of well-being (measured in utilities), which may be called "*welfare egalitarianism*". Finally, radical or pure egalitarianism has to be distinguished from moderate or pluralist egalitarianism, where *radical or pure egalitarianism* strives for equality only, whereas moderate or pluralist egalitarianism has additional aims beyond

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<sup>11</sup> Loosely speaking, one could say: Intrinsic egalitarianism strives for some distributive pattern as its final end. But this is misleading because it may be misunderstood as if egalitarians were not interested in improving well-being, remedy human suffering etc. but in realizing abstract distributive patterns. (Some anti-egalitarians are too happy to misunderstand egalitarianism in this way [cf. e.g. Hausman 2001, 2 f.].) But, of course, the overwhelming majority of egalitarians is intrinsically interested in the latter things. However it is empirically impossible to guarantee the hypothetic individual optimum to everybody; there are distribution conflicts; and it has to be decided who should get how much help, who should help how much etc. Egalitarians prefer to take these decisions according to egalitarian value functions - whatever their reasons are for this preference. So egalitarians are intrinsically striving for improving welfare according to a certain distributive pattern.

that; usually it is maximizing total utility. Pure egalitarianism is *radical* in the sense of being prepared to accept violations of the Pareto principle for equality's sake. In particular it may prefer to reduce the well-being of somebody very well off without any compensation because this reduces inequality. Most people find such a preference for *levelling down* objectionable or even disgusting. Therefore, (welfare) egalitarians nearly exclusively are pluralists and moderate in that respect that they stick to the Pareto principle. I.e. they regard every increase of somebody's utility (which leaves the others unaffected) as a moral improvement - even if it goes to the person best off. [Cf. Parfit 1997, 218.]

What then is the characteristic feature of welfare egalitarianism? *Welfare egalitarianism* as such (i.e. pure egalitarianism in a pure way and pluralist egalitarianism besides other aims) strives for equality of well-being, it seeks to "equalize utilities" [Atkinson / Stiglitz 1980, 404 12]. Formulated in a negative way: Egalitarianism as such values deviations from a (hypothetical) state of equality as negative, the bigger these deviations are the more negative (more than proportionally). This holds for downward deviations as well as, *ceteris paribus*, for upward deviations, which in this respect are valued symmetrically, i.e. equally negative, depending on the absolute value of the deviation only.<sup>13</sup> This symmetry is essential for egalitarianism because if somebody is exclusively interested in equality the direction of deviation from equality should not matter; and if he or she is interested in equality only among other aspects the direction of deviation should not matter for the egalitarian aspects of his valuations. The latter case obviously does not rule out to include such other aspects in the moral valuation, which together with the egalitarian aspect lead to altogether fulfilling the Pareto principle for upward deviations from equality. To sum up, egalitarianism here is characterized by two conditions, the *symmetry condition*, which says that upward and downward deviation from some middle must be valued equally negative, and the *increasing weight condition*, which says that greater deviations should be valued increasingly stronger. - Egalitarian valuations are more heterogeneous than prioritarian valuations, e.g. they are not always representable by one-adic value functions. Therefore and because my primary concern is prioritarianism and not egalitarianism, I (have to) leave the characterization of egalitarianism at that.

Some first implications of the characterization of 'egalitarianism' just given are these.

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12 Atkinson and Stiglitz distinguish between a narrow and a broad notion of 'egalitarianism' as has been done here. The quotation just given, of course, refers to the narrow sense, whereas egalitarianism in the broad sense for them includes e.g. even leximin [Atkinson / Stiglitz 1980, 404].

13 Temkin as well as Parfit write that egalitarians believe it is bad for some to be worse off than others through no fault of their own [Temkin 1993, 138; Parfit 1997, 204]. They do not mention upward deviation probably because they deal mainly with moderate egalitarianism, which (as a consequence of the Pareto principle and non-egalitarian considerations) values upward deviations always positively. But if one speaks of pure egalitarianism or only of the egalitarian part of moderate egalitarianism upward deviations from equality should be treated like downward deviations.

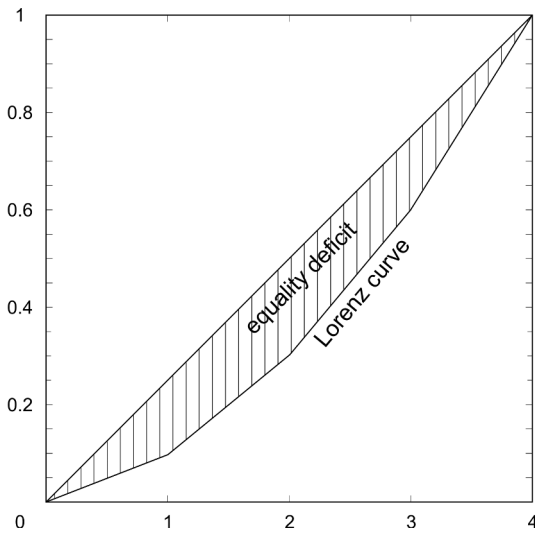
1. Because egalitarianism is interested in diminishing, or better: completely abolish, interpersonal differences it is universalistic or symmetrical - which does not exclude, though, that the realm of equality is restricted in a parochialist fashion.
2. Radical egalitarianism violates the Pareto principle (i.e. its value function is not monotonously increasing), whereas moderate egalitarianism has been defined as fulfilling the Pareto principle. But as we will see a bit later, even pluralist egalitarianism, e.g. versions that use the standard deviation as their inequality measure, may at least tend to violate the Pareto principle for very high utility levels [Trapp 1988, 358]. This is a consequence of the increasing weight for great deviations.
3. Egalitarianism has to fulfil the Pigou-Dalton condition, but for different reasons than prioritarianism. *Prima facie* the Pigou-Dalton condition seems to even have a prioritarian background because it says that transfers from the better off to those worse off (in certain limits) shall be valued positively, and this may be taken as expressing the fundamental prioritarian idea: The positive valuation follows from the feature that the weight attributed to alterations of the better offs' utilities is lower than that attributed to alterations for the worse offs. But there is an egalitarian interpretation of the Pigou-Dalton condition as well: If the redistribution goes from someone above the middle to someone below the middle both are approaching the middle so that inequality obviously is diminished. If the redistribution takes place within the same side of the middle one person is approaching the middle to the same degree as the other is moving away from it so that nothing seems to be gained. But remember that deviations from the middle are negatively weighted the greater they are. Therefore the approaching movement, which is always that of the person in the extremest position, is valued higher than the deviating movement, which is that of the person nearer to the middle. (Reasoning strictly, more cases have to be dealt with, namely where one or both persons change from one to the other side of the middle. But the result is always the same.) Technically the Pigou-Dalton condition is fulfilled by prioritarianism because of the concavity of its value function  $VP$ . The argument just developed implies that all forms of egalitarianism (in the sense defined above) must be concave too - though they may not be representable via a one-adic concave egalitarian value function  $VE$  so that only the more general definition of 'concavity' (cf. above, fn 3) may be applied to them.
4. Because 'equality' is essentially a comparative notion, egalitarian valuation is comparative; it assesses interpersonal differences in well-being [Parfit 1995, 23; Temkin 1993, 138]. Therefore, egalitarian welfare functions are not additively separable [Broome 2002, 2; Klint Jensen 2002, 4]. And this implies that the moral valuation of the well-being of subgroups is not independent of the well-being of the rest of the group. The same change occurring in some subgroup may be valued differently if the cardinality or the level of well-being of the rest of the group is different. [Klint Jensen 2002, 18 f.] (Technically, of course, this is an irksome quality of egalitarian valuation. But this inconvenience does not amount to a moral objection.)
5. Because of the interpersonal utility comparisons egalitarianism requires utility levels to be interpersonally comparable. But because the specific aim of egalitarianism consists only in these comparisons it does not require the determination of standardized utility measure - as prioritarianism does. Every interpersonally comparable utility measure will do equally well.

Which common welfare functions are egalitarian, according to the definition given above? All welfare functions based on the variance ( $\text{VARIANCE} = 1/n \cdot \sum_{i=1}^n (u_\mu - u_i)^2$ , with  $u_\mu$  being the mean utility level), the standard deviation ( $\sigma_u = \sqrt{1/n \cdot \sum_{i=1}^n (u_\mu - u_i)^2}$ ) or the coefficient of variation ( $\text{COV} = 1/u_\mu \cdot \sqrt{1/n \cdot \sum_{i=1}^n (u_\mu - u_i)^2}$ ) are paradigm cases of egalitarianism. (This class of egalitarian welfare functions may be extended by including functions that instead of squaring and extracting the square root use other exponents  $p$  and other bases  $p$  of the root with  $p > 1$  (in this case the inner brackets have to be replaced by the sign for the absolute value).<sup>14</sup>) Rescher e.g. has proposed a welfare function, which he calls the "*effective-average-principle*" and which is defined as the average utility minus half the standard deviation, i.e.

$$VET_{SD}(a) = u_\mu - 1/2 \cdot \sqrt{1/n \cdot \sum_i (u_\mu - u_i)^2},$$

again with  $u_\mu$  as the mean utility level. This is a pluralist version of egalitarianism with mean utility as one aim and avoiding inequality, measured via the standard deviation, as the other aim. So, subtracting the standard deviation is the egalitarian part of this function. Now if we look to the inner parenthesis of  $VET_{SD}$ , i.e.  $u_\mu - u_i$ , it can easily be seen that upward and downward deviations of the same absolute value from the mean are valued equally negative (the squaring procedure makes their signs equal). So the symmetry condition is fulfilled. In addition, the squaring procedure implies that the weight attributed to deviations is increasing with the deviation's absolute value (a deviation of 0.1 is weighted by 0.1, a deviation of 0.2 is weighted by 0.2 etc.). This means that the condition of over-proportional weighting of deviations is fulfilled too.

Fig. 2: Lorenz curve: x-axis: persons  $i$ ; y-axis: accumulated  $u$



Another group of welfare functions fulfilling the above definition of egalitarianism are those developed out of the Gini-coefficient. The Gini-coefficient can be defined in various ways.

<sup>14</sup> Sen (and other authors) have objected to squaring and the square root that it is arbitrary [Sen 1973, 28; Temkin 1993, 123]. This arbitrariness can be avoided in the way just proposed, i.e. by admitting various exponents  $p$  and bases  $p$  of the root, which express one's inequality aversion.  $p=1$  then is the limiting case, which applied to the coefficient of variation generates the relative mean deviation:  $IE_{RMD} = 1/nu_\mu \cdot \sum_{i=1}^n |u_\mu - u_i|$ . (For why this is a limiting case only, see below.) Higher  $p$ s express increasing degrees of inequality aversion.

The best (because most intuitive) definition to start with, probably, is that referring to the Lorenz curve (cf. figure 2).<sup>15</sup> For the discrete case the definition is:

$GINI_1 := [(n/2) \cdot (n+1) \cdot u_\mu - \sum_{i=1}^n \sum_{j=1}^i u_j] / [(n/2) \cdot (n+1) \cdot u_\mu]$ , where the  $u_j$  are ordered *increasingly*, from the bottom to the top:  $u_1 \leq u_2 \leq \dots \leq u_n$ .

$\sum_{j=1}^i u_j$  is the sum of all utilities up to utility position  $i$ ; so it represents the Lorenz curve at point  $i$ .  $\sum_{i=1}^n \sum_{j=1}^i u_j$  represents the sum of all these points, i.e. something similar to the integral under the Lorenz curve. The first half of the numerator and the denominator instead represent something like the integral under the curve of complete equality. The difference in the numerator then represents something like what is missing to complete equality, which then, by the division, is compared to complete equality.  $GINI_1$  now is equal to several other expressions.

$GINI_1 = GINI_2 = 1 + (1/n) - (2/(n^2 u_\mu)) \cdot [u_1 + 2u_2 + \dots + nu_n]$ , where the  $u_i$  this time are ordered *decreasingly*:  $u_1 \geq u_2 \geq \dots \geq u_n$

$= 1 + (1/n) - (2/(n^2 u_\mu)) \cdot \sum_i i \cdot u_i =$

$GINI_3 = (1/(2n^2 u_\mu)) \cdot \sum_{i=1}^n \sum_{j=1}^n |u_i - u_j|$ .

The kernel of  $GINI_2$  is:  $\sum_i i \cdot u_i$ , i.e. each individual's utility level is multiplied by its rank, counting from the top. The kernel of  $GINI_3$  is the double sum at the end (the factor before this double sum serves only for creating the right order of magnitude, i.e. a number between 0 and 1). This double sum means that the differences between all utility values are summed up (more exactly: they are summed up twice because for all  $u_k$  and  $u_l$   $|u_k - u_l|$  as well as  $|u_l - u_k|$  end up in the sum). Measuring inequality by summing up all interpersonal differences has been found attractive by many egalitarians because it directly expresses one conception of inequality. An egalitarian welfare function based on the  $GINI_3$  e.g. is Trapp's [1988; 1990] "*utilitarianism incorporating justice*", the value function of which is:

$VET_{UJ}(a) = u_\mu - (d/n^2) \sum_{i=1}^n \sum_{j=i+1}^n (u_i - u_j)$ , where the  $u_i$  again are ordered decreasingly:  $u_1 \geq u_2 \geq \dots \geq u_n$ , and where  $d \in [0, 1]$  [cf. Trapp 1988, 356; 1990, 365].<sup>16</sup>

$d$  is a parameter expressing one's inequality aversion, with  $d=0$  representing complete absence of some inequality aversion and turning  $VET_{UJ}$  into average level utilitarianism.  $d=1$ , the degree preferred by Trapp himself, is the highest degree of inequality aversion that still guarantees fulfilment of the Pareto principle. Division by  $n^2$  again serves only for creating the right order of

<sup>15</sup> The *continuous Lorenz curve* specifies how many percent of a total - in our case the total of utilities - belong to the lowest  $i$  per cent of the population. So both axes measure in percent. The continuous Lorenz curve always runs from  $\langle 0, 0 \rangle$  to  $\langle 1, 1 \rangle$ . For a complete equal distribution, where the lowest  $i$  per cent of the population accumulate always  $i$  per cent of the total utility, the Lorenz curve is straight. The more unequal the distribution is the more the Lorenz curve is bulged towards point  $\langle 1, 0 \rangle$ . - For the discrete case the x-axis represents the single persons from 1 to  $n$  with increasing utility levels. And the curve specifies the sum of the utilities for the lowest  $i$  people. So its value for  $x=i$  is:  $u_1 + u_2 + \dots + u_i$ . - The *Gini coefficient* is defined as the area between the Lorenz curve and the line of equal distribution - which represents what is missing to equality - divided by the area below the line of complete equality.

<sup>16</sup> Trapp's criterion, in addition to what just has been reproduced, contains factors expressing the individuals' merits. Because this has little to do with the question of equality I have omitted that here. Furthermore, Trapp's formula has been technically transformed here a bit for approaching it to the mode of representation used so far.

magnitude. The final double sum is changed as compared to that of  $GINI_3$  in that all the differences enter only once into the sum; and because the  $u_i$  are ordered decreasingly the differences are always positive (so that the sign for the absolute value could be eliminated); the double sum, i.e. the kernel of Trapp's inequality measure, then means:  $IE_{UJ} = \sum_{i=1}^n \sum_{j=i+1}^n (u_i - u_j) =$

$$(u_1 - u_2) + (u_1 - u_3) + (u_1 - u_4) + (u_1 - u_5) + \dots + (u_1 - u_n) +$$

$$(u_2 - u_3) + (u_2 - u_4) + (u_2 - u_5) + \dots + (u_2 - u_n) +$$

$$(u_3 - u_4) + (u_3 - u_5) + \dots + (u_3 - u_n) +$$

$$\text{etc.} +$$

$$(u_{n-1} - u_n).$$

Now  $IE_{UJ}$  is equal to:

$$IE_{UJf} = \sum_{i \leq (n+1)/2} (n+1-2i) \cdot d_i + \sum_{i > (n+1)/2} (2i-n-1) \cdot d_i, \text{ where } d_i \text{ is the distance of } u_i \text{ to the median }^{17} u_m,$$

i.e.:  $d_i := |u_m - u_i|$ .

The proof is given in the appendix. The left sum of  $IE_{UJf}$  (with  $i$  ranging from 1 to  $(n+1)/2$  included) deals with the utility levels on or above the median. The respective distances  $d_i$  to the median are multiplied by  $n+1-2i$ . For, let us say,  $n=8$  the left sum is equal to:  $7d_1+5d_2+3d_3+1d_4$ . The right sum of  $IE_{UJf}$  (with  $i$  ranging from  $(n+1)/2$  excluded to  $n$ ) deals with the utility levels below the median. The respective distances  $d_i$  to the median now are multiplied by  $2i-n-1$ , which is the same factor as in the left sum but multiplied by  $-1$ . For, again,  $n=8$  the right sum is equal to:  $1d_5+3d_6+5d_7+7d_8$ . (The respective expressions for odd numbers  $n$ , e.g.  $n=7$ , are: left sum:  $6d_1+4d_2+2d_3+0d_4$ ; right sum:  $2d_5+4d_6+6d_7$ .  $d_4=0$  because in this case  $u_4=u_m$ .) This means the inequality measure  $IE_{UJ}$  treats the median  $u_m$  as the middle and weights all the distances  $d_i$  of the individual utility levels  $u_i$  from that middle according to some rank-order: For each position a realized utility level is farther away from the middle the summand 2 has to be added to the weight with which the distance  $d_i$  has to be multiplied, for both sides symmetrically. This corresponds to the above definition of 'welfare egalitarianism': 1. Equal deviations from the middle are valued symmetrically, and 2. they are weighted increasingly.<sup>18, 19</sup>

Some welfare functions or measures of inequality that are often held to be egalitarian, according to the definition given above are not egalitarian.

The *relative mean deviation* is an inequality measure defined as:  $IE_{RMD} = (1/nu_\mu) \cdot \sum_i |u_\mu - u_i|$ . It could be transformed trivially into a moral value function by multiplying it by  $u_\mu$  and subtracting the result from  $u_\mu$  itself ( $V_{RMD} := u_\mu - (1/n) \cdot \sum_i |u_\mu - u_i|$ ). It is only a limiting case of egalitarianism

<sup>17</sup> The *median* level of welfare roughly is that level where there are the same number of people at or above it, as at or below it. For odd numbers  $n$  of ordered utilities, the median  $u_m = u_{(n+1)/2}$  (e.g. for  $n=7$   $u_m = u_4$ , which is in the middle of the row). For even numbers  $n$  of ordered utilities, the median is the arithmetic mean of the two middle utilities:  $u_m = (1/2) \cdot (u_{n/2} + u_{(n/2)+1})$  (e.g. for  $n=8$   $u_m = (1/2) \cdot (u_4 + u_5)$ ).

<sup>18</sup> Some people have objected to rank-order dependant weighting that it is too arbitrary because the relative weight attributed to two positions depends on how many people lie between these positions [Sen 1973, 33]. I think this is a strong criticism. But discussing the merits of various forms of egalitarianism is not the scope of this paper.

<sup>19</sup> A further egalitarian (in the strong sense, including symmetry) welfare function has been proposed by von Kutschera [1982, 141 f.].



because, though upward and downward deviations from the mean are valued symmetrically, these deviations are not valued more than proportionally, i.e. the increasing weight condition is violated. (This then leads to the well-known problem that redistributions on the same side of the mean, e.g. transfers from somebody a bit under the mean to the worst off, cannot be valued as affecting inequality [Sen 1973, 26]; this furthermore implies that the Pigou-Dalton condition is violated [Temkin 1993, 122].) Though the relative mean deviation obviously is egalitarian in spirit its inadequacy in not representing many changes in the degree of inequality is so striking that probably no informed person will defend it as a morally adequate inequality measure. (Sen e.g. in his discussion of inequality measures calls it "a non-starter" [Sen 1973, 31].) Therefore, there is no need to broaden our definition of 'egalitarianism' for including value functions based on the relative mean deviation.

The *standard deviation of the logarithm* is an inequality measure defined as:  $IE_{SDL} := \sqrt{[(1/n) \cdot \sum_i (\log u_i - \log u_\mu)^2]}$ . Whilst the relative mean deviation did not weight deviations from the middle increasingly, i.e. did not fulfil the second condition for egalitarianism (increasing weight condition), the standard deviation of the logarithm does not fulfil the first condition (symmetry condition), i.e. does not treat upward and downward deviations from the middle symmetrically. Rather it weights downward deviations stronger.<sup>20</sup> This may even sound a bit prioritarian; but it is surely not a welfare egalitarian concern.<sup>21</sup>

Atkinson, following Dalton, has introduced an 'inequality measure' for incomes, which is defined as:  $IE_{AT} = 1 - (y_{EDE}/y_\mu)$ , where  $y_\mu$  is the real mean income and  $y_{EDE}$  is the hypothetical equal income for all that would produce the same total utility as the actual income distribution [Atkinson 1970, 250]. Though speaking of equal distributions, this measure, obviously, is not apt as a basis for some welfare egalitarianism. First, it directly deals with income instead of well-being and, second, upward and downward deviations from the mean again are weighted differently, namely via a concave utility function over income. But there is a deeper reason why one should not try to include a welfare function based on Atkinson's inequality measure in the extension of 'welfare egalitarianism'. The underlying welfare conception of  $IE_{AT}$  is *utilitarian*;<sup>22</sup> the aim is to maximize total utility. Equality of income is only a means to that final end - based on the well-known idea

20 Take e.g.  $u_\mu=0.5$  and consider  $u_1=0.7$  and  $u_2=0.3$ , which are equally distant from  $u_\mu$ . The inequality contribution of  $u_1$  then is  $(-0.301 - (-0.155))^2 = 0.0213$ , whereas the inequality contribution of  $u_2$  is  $(-0.301 - (-0.523))^2 = 0.0493$ .

21 Weighting upward deviations from the mean less may make *some* sense (I do not say "sufficient sense") if one wants to assess inequality of *income* because such deviations may reach orders of magnitude of 100.000 times the mean income. Wanting to satisfy the Pareto principle, the Pigou-Dalton condition, the symmetry condition and the increasing weight condition may lead to inequality measures that are very insensible to changes occurring close to the middle. But this argument clearly does not hold for inequality measures over *utility* because the highest upward deviations of utility levels are far lower than 10 times the mean.

22 Atkinson departs from this utilitarian origin though by not using an *empirically* justified utility function over income but by, unexpectedly, choosing such utility functions on *normative* grounds instead [cf. Atkinson 1970, 251; 257]. Introducing these normative features is not only *ad hoc* but leads also to a confusing and arbitrary mixing of aims of measurement, namely a utilitarian measure of the inefficiency of the income distribution and a 'prioritarian' measure of income equity.

that with interpersonally equal utility functions over income and a fix total income total utility would be maximized by an equal income distribution. So  $IE_{AT}$  is not an inequality measure at all, not even for income, rather it is a *utilitarian inefficiency measure* for income distributions.

### 2.3. Differences between Prioritarianism and Egalitarianism

The differences between prioritarianism and welfare egalitarianism elaborated so far are: 1. Prioritarianism has to fulfil the Pareto principle whereas radical egalitarianism violates it. 2. Prioritarian valuations must be additively separable whereas egalitarian valuations cannot be so because they necessarily imply comparisons. 3. Prioritarian valuations must use standardized utilities whereas for egalitarian valuations interpersonally comparable utilities are sufficient. The first difference, of course, does not hold with respect to moderate egalitarianism, and the third seems to be rather technical as to be of great moral importance. So we are reduced to the second difference, which has already been used by other theoreticians to define the difference between prioritarianism and egalitarianism [Broome 2002, 2; Klint Jensen 2002, 4]. This difference is important, as Broome underlines, because it expresses the difference between an interest in improving people's lot according to their absolute level of well-being versus an interest in interpersonal comparisons and diminishing interpersonal differences. But it would be of no practical importance if for each prioritarian value function we could find an egalitarian one leading always to the same rankings - as Fleurbaey [2002, 6 f.] asserts. And in fact it has been difficult to find examples where (all) prioritarian and (all) egalitarian valuations must lead to diverging rankings. In particular moderate egalitarianism with concave value functions can be very near to prioritarianism. So to find a difference between these systems is the real challenge.

However, the definitions of 'prioritarianism' and 'egalitarianism' given above contain a still more important difference. It has not been manifest so far because the definition of 'egalitarianism', intended to be open to technically rather divergent criteria, was not mathematically as elaborated as that of 'prioritarianism'. But focussing our attention now on the egalitarian criteria technically nearest to prioritarian criteria, the lower mathematical elaboration can be made up for. Those criteria come near to the form of Rescher's effective-average-principle, which was:

$$VET_{SD}(a) = u_{\mu} - 1/2 \cdot \sqrt{[1/n \cdot \sum_i (u_{\mu} - u_i)^2]}.$$

If we omit the procedure of extracting the root and substitute the first occurrence of  $u_{\mu}$  by its definition (i.e.  $1/n \cdot \sum_i u_i$ ) we get:

$$VET_{VAR2}(a) = 1/n \cdot \sum_i u_i - (1/2n) \cdot \sum_i (u_{\mu} - u_i)^2,$$

which can be rearranged as:

$$VET_{VAR2}(a) = 1/n \cdot \sum_i [u_i - 1/2 \cdot (u_{\mu} - u_i)^2].$$

This function is not really additively separable because it compares any single utility level  $u_i$  with  $u_{\mu}$ . But keeping  $u_{\mu}$  fix, e.g. as  $u_{\mu}=0.5$ , and omitting the initial division by  $n$ , we can obtain a function of the contribution of an individual utility level to the egalitarian welfare, and this is:

$$VE_{VAR2}(u) = u - 1/2 \cdot (u_{\mu} - u)^2.$$

In order to standardize this function in the interval  $[0, 1]$  we can add  $-1/2 \cdot u_{\mu}^2 (= 1/8)$ , obtaining:

$$VE_{VARS2}(u) = u - 1/2 \cdot (u_{\mu} - u)^2 + 1/8 \text{ (cf. figure 3).}$$

This is a concave function with  $1/2 \cdot (u_{\mu} - u)^2$  being the inequality contribution of  $u$ , which has to be subtracted from  $u$  itself; this  $u$  is the second, utilitarian component of the criterion, turning it into a pluralistic and moderate version of egalitarianism, which (in a rather wide interval of  $u$ ) fulfils the Pareto principle. Subtracting the inequality contribution of  $u$  leads to fulfilling the two conditions for egalitarianism: Upward and downward deviations from  $u_{\mu}$  are treated symmetrically and the squaring procedure means giving increasing weight to greater deviations.

Fig. 3a: Egalitarian value function  $VE_{VARS2}$  based on variation, individual contribution  $u_{\mu}$  fixed ( $u_{\mu}=0.5$ )

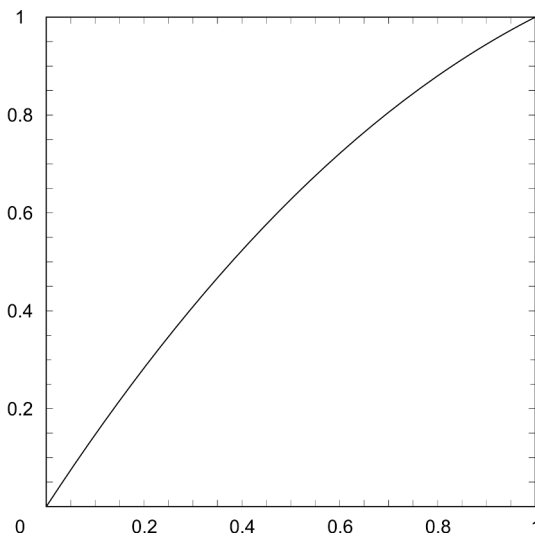
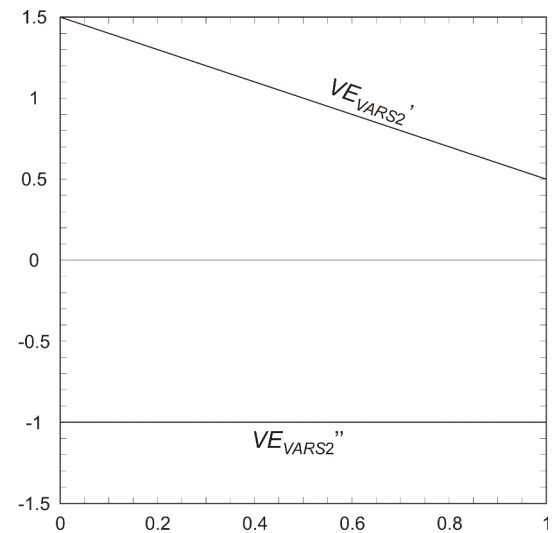


Fig. 3b: Derivations of egalitarian value function based on variation, individual contribution  $u_{\mu}$  fixed ( $u_{\mu}=0.5$ )



The mathematical implications of the symmetry and the increasing weight condition can best be explained if we focus for a moment on this inequality component:

$$IC_{VARS2}(u) = -1/2 \cdot (u_{\mu} - u)^2 + 1/8.$$

And the general form of this function (cf. footnote 14) is:

$$IC_{VARSp}(u) = -a \cdot |u_{\mu} - u|^p + a \cdot u_{\mu}^p, \text{ with } p > 1 \text{ and } a > 0 \text{ (the graphs for } a=1/2 \text{ and } p=1.5, p=2, p=3 \text{ are represented in figure 4).}$$

The function  $IC_{VARS2}$  (but the functions  $IC_{VARSp}$  as well) forms a little hill having its maximum in  $u_{\mu}$ , this implies  $IC_{VARS}'(u_{\mu})=0$ . The hill is axially symmetric with respect to the axis  $u=u_{\mu}$ , which is necessary for fulfilling the symmetry condition, i.e.  $IC_{VARS}(u_{\mu}-d) = IC_{VARS}(u_{\mu}+d)$ . And the hill is concave, which is necessary for fulfilling the increasing weight condition. This concavity of  $IC_{VARS}$  is equivalent to the second derivation being below 0 ( $IC_{VARS}'' < 0$  throughout). That  $IC_{VARS}$  is axially symmetric with respect to the axis  $u=u_{\mu}$  now implies: 1. that its first derivation  $IC_{VARS}'$  must be point-symmetric with respect to the point  $\langle u_{\mu}, 0 \rangle$ , i.e.  $IC_{VARS}'(u_{\mu}-d) = -IC_{VARS}'(u_{\mu}+d)$ , 2. that its second derivation  $IC_{VARS}''$  must be axially symmetric with respect to the axis  $u=u_{\mu}$ , i.e.  $IC_{VARS}''(u_{\mu}-d) = IC_{VARS}''(u_{\mu}+d)$ , and 3. that its third derivation  $IC_{VARS}'''$  again must be point-symmetric with respect to the point  $\langle u_{\mu}, 0 \rangle$ , i.e.  $IC_{VARS}'''(u_{\mu}-d) = -IC_{VARS}'''(u_{\mu}+d)$ . But if the first derivation  $IC_{VARS}'$  is point-symmetric with respect to  $\langle u_{\mu}, 0 \rangle$  it cannot be strictly convex throughout. Either  $IC_{VARS}'$  is

(quasi-)convex until  $u_\mu$  and then (quasi-)concave for  $u > u_\mu$ , or the other way round: first (quasi-)concave and then (quasi-)convex.<sup>23</sup> This (for three times differentiable functions like  $IC_{VARS}$ ) implies for the third derivation  $IC_{VARS}'''$ : Either for  $u \leq u_\mu$   $IC_{VARS}'''(u) \geq 0$  and for  $u > u_\mu$   $IC_{VARS}'''(u) \leq 0$ , or the other way round: for  $u \leq u_\mu$   $IC_{VARS}'''(u) \leq 0$  and for  $u > u_\mu$   $IC_{VARS}'''(u) \geq 0$ . Summarizing and generalizing for egalitarian functions  $IC$  of individual inequality contributions, we have the following implications:

0.  $IC(u)$  is axially symmetric with respect to  $u = u_\mu$ .

1.  $IC'(u_\mu) = 0$ ; for  $u < u_\mu$ :  $IC'(u) > 0$ ; for  $u > u_\mu$ :  $IC'(u) < 0$ ; and  $IC'(u)$  is point-symmetrical with respect to  $\langle u_\mu, 0 \rangle$ .

2.  $IC''(u) < 0$  throughout (= concavity of  $IC$ );  $IC''(u)$  is axially symmetrical with respect to  $u = u_\mu$ .

3.  $IC'''(u)$  is point-symmetrical with respect to  $\langle u_\mu, 0 \rangle$ ; either for  $u \leq u_\mu$   $IC'''(u) \leq 0$  (quasi-concavity of  $IC'$ ) and for  $u \geq u_\mu$   $IC'''(u) \geq 0$  (quasi-convexity of  $IC'$ ), or for  $u \leq u_\mu$   $IC'''(u) \geq 0$  (quasi-convexity of  $IC'$ ) and for  $u \geq u_\mu$   $IC'''(u) \leq 0$  (quasi-concavity of  $IC'$ ).

Fig. 4a: Inequality contribution of individual utility ( $u_\mu$  fixed ( $u_\mu = 0.5$ )):  $IC_{VARS1.5}(u)$ ,  $IC_{VARS2}(u)$ ,  $IC_{VARS3}(u)$

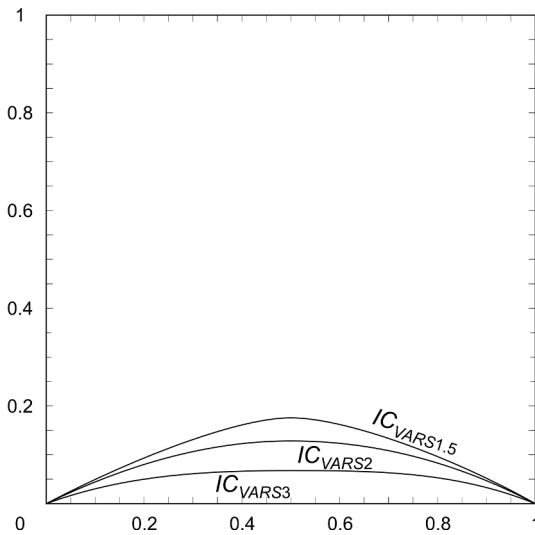
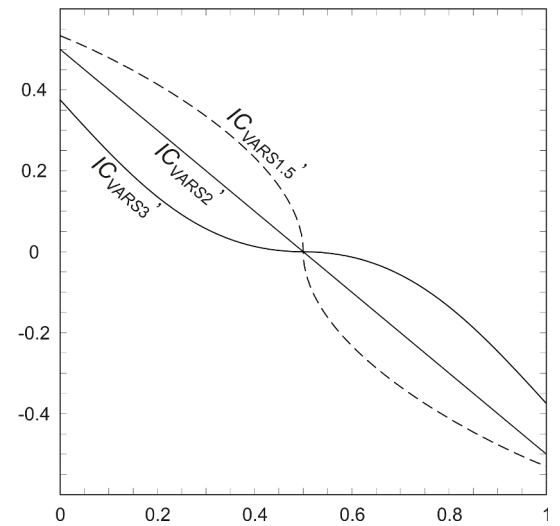


Fig. 4b: Derivations of inequality contributions of individual utilities:  $IC_{VARS1.5}'(u)$ ,  $IC_{VARS2}'(u)$ ,  $IC_{VARS3}'(u)$



So far we have considered only the inequality contribution  $IC_{VAR(S)}$  of the individual utility levels to the egalitarian valuation. But this is not yet comparable to the prioritarian value function  $VP$ , only the function of the complete contribution  $VE_{VAR(S)}$  of individual utility levels is comparable to  $VP$ .  $VE_{VARS}$  differs from  $IC_{VARS}$  by including the utilitarian aspect too, i.e. by adding  $u$  to  $IC_{VARS}(u)$ :  $VE_{VARS}(u) = IC_{VARS}(u) + u$ . This addition makes  $VE_{VARS}(u)$  monotonously increasing (at least for a wide range of  $u$  (cf. below)). But it has little impact on the derivations. The first

23

Both cases are possible. If we take for example a generalized version of  $IC_{VARS}$ , namely:  $IC_{VARS_p}(u) = -a \cdot |u_\mu - u|^p + a \cdot u_\mu^p$ , with  $p > 1$  and  $a > 0$ , then for  $1 < p < 2$   $IC_{VARS_p}'$  is strictly concave up to  $u_\mu$  and strictly convex for higher  $u$ , and for  $p > 2$   $IC_{VARS_p}'$  is strictly convex up to  $u_\mu$  and strictly concave for higher  $u$ . For  $p = 2$   $IC_{VARS_p}'$  is linear, which may be described as being first quasi-concave and then quasi-convex or the other way round or quasi-concave throughout or quasi-convex throughout. But in any case it is not strictly concave.

derivation  $VE_{VARS}'$  is raised with respect to  $IC_{VARS}'$  by one unit. And the second and third derivation remain unchanged, i.e.  $VE''=IC''$  and  $VE'''=IC'''$ . So we get the following conditions (cf. figure 3):

1.  $VE'(u_\mu)=1$ ; for  $u < u_\mu$   $IC'(u) > 1$ ; for  $u > u_\mu$   $VE'(u) < 1$ ; and  $VE'(u)$  is point-symmetrical with respect to  $\langle u_\mu, 1 \rangle$ .
2.  $VE''(u) < 0$  throughout (= concavity of  $VE$ );  $VE''(u)$  is axially symmetrical with respect to  $u=u_\mu$ .
3.  $VE'''(u)$  is point-symmetrical with respect to  $\langle u_\mu, 0 \rangle$ ; either for  $u \leq u_\mu$   $VE'''(u) \leq 0$  (quasi-concavity of  $VE'$ ) and for  $u \geq u_\mu$   $VE'''(u) \geq 0$  (quasi-convexity of  $VE'$ ), or for  $u \leq u_\mu$   $VE'''(u) \geq 0$  (quasi-convexity of  $VE'$ ) and for  $u \geq u_\mu$   $VE'''(u) \leq 0$  (quasi-concavity of  $VE'$ ).

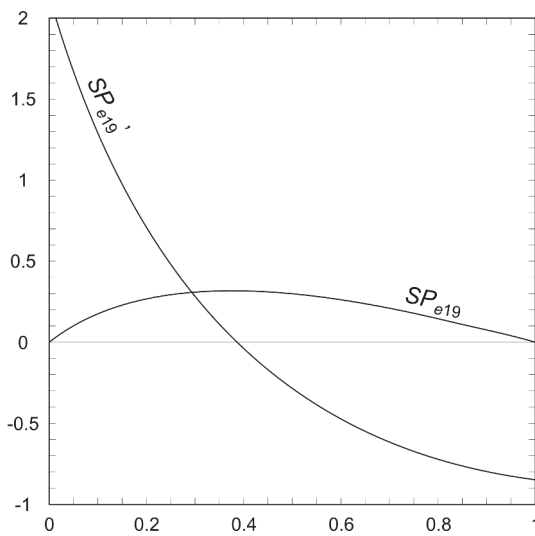
The first condition implies that the value function  $VE$  need not be monotonously increasing throughout: For high values of  $u$  the subtraction for their inequality contribution may grow faster than the growth in  $u$  itself so that  $VE'$  becomes negative and  $VE$  itself declining, thus perhaps violating the Pareto-principle.<sup>24</sup> But moderate egalitarians will try to avoid this consequence by choosing the right parameters. So the endeavour to distinguish between prioritarianism and egalitarianism should not stress this point too much. - The really striking difference to prioritarianism then is entailed in condition 3. The respective condition for prioritarianism says:  $VP'''(u) > 0$  for all  $u$  (cf. above P4.3), i.e. the first derivation  $VP'$  of the value function  $VP$  has to be convex throughout, which had been identified as *the* characterizing feature of prioritarianism. Expressed intuitively it means that importance attributed to further improvements of the well-being is decreasing continuously and smoothly with increasing initial well-being but always remaining positive; so prioritarianism welcomes all improvements and is not hostile to any initial well-being level but gives improvements less and less importance. An egalitarian weighting function  $VE'$  instead that first decreases convexly and after  $u_\mu$  concavely means that initial utility levels above  $u_\mu$  are penalized in a certain way in that the weight given to improvements for such persons is declining faster and faster. And this can be understood only from the egalitarian perspective, namely that upward deviations from the middle get some negative weighting the greater they are. An egalitarian weighting function  $VE'$  of the other type, i.e. which first decreases concavely and beyond  $u_\mu$  convexly from a prioritarian perspective is even stranger. It resembles a two-step stair descending to the right. And this means up to a certain level of well-being below the mean improvements are weighted highly and nearly equally. Beyond that level the importance given to further improvements is declining rapidly and from a certain level above mean onwards it remains nearly stable. The limiting case is that society is divided in two groups, the worse-offs and the better-offs, which get different degrees of attention, where the cutting line between them, i.e. the mean  $u_\mu$  seems to be arbitrary from an absolute point of view.

(A further respect in that prioritarian and egalitarian functions differ is this. The analogue to the inequality contribution  $IC_{VARS_p}$  in prioritarianism is the function we get by detracting  $u$  from the prioritarian value function  $VP$ :  $SP(u) := VP(u) - u$  (cf. figure 5). This function may be called the

<sup>24</sup> Increasing  $u$  make the mean  $u_\mu$  increasing, too. This is not reflected in  $VE$ . Therefore, a declining egalitarian value function  $VE$  does not immediately translate into violations of the Pareto-principle. But with high  $n$  and a stronger decline they do.

"prioritarian surplus" function (i.e. the surplus beyond the utilitarian value). From the prioritarian point of view this surplus function has no intuitive sense because the prioritarian valuation is not composed of the utilitarian value plus some surplus; introducing the surplus function  $SP$  serves only for reasons of comparison. In contrast to the symmetrical egalitarian function of the inequality contribution  $IC_{VARSP}$ , this prioritarian surplus function  $SP$  is left-skewed. And it must be so because left-skewedness means that the slope of this curve is decreasing more and more slowly. And this again is equivalent to  $SP'$  decreasing in a convex fashion. But  $SP'$  is only the result of diminishing  $VP'$  by one unit ( $SP'(u) = VP'(u) - 1$ ), and the convexity of  $VP'$  was the characteristic feature of prioritarianism. So left-skewedness of  $SP$  is an immediate consequence of the characteristic prioritarian feature.)

Fig. 5: Prioritarian surplus function:  $SP(u) := VP(u) - u$



The decreasing weight of further improvements (priority to the worse-off) versus symmetrical discounting of deviations from the middle leads to different rankings by even rather similar prioritarian versus egalitarian value functions. Consider the following utility distributions:

$$a = \langle 0.75, 0.75, 0 \rangle,$$

$$b = \langle 1, 0.25, 0.25 \rangle \text{ [cf. Lumer 2000, 631].}$$

$a$  and  $b$  have the same mean, namely  $u_{\mu} = 0.5$ . They are constructed in such a way that in  $a$  there are two upper deviations of 0.25 and one lower deviation of 0.5 from this middle, whereas in  $b$  these deviations are exactly reversed: two lower deviations of 0.25 and one upper deviation of 0.5 from the middle. In addition, though the median of  $a$  is 0.75 and that of  $b$  is 0.25, in both options the deviations from these medians again are symmetrical: In  $a$  there is exactly one (downward) deviation of 0.75 from the median 0.75, and in  $b$  there is exactly one (upward) deviation of 0.75 from the median 0.25. Therefore, all egalitarian value functions because of their fulfilling the symmetry condition have to value  $a$  and  $b$  as equivalent. Prioritarian valuations, on the other hand, prefer  $b$  to  $a$ . In going from  $a$  to  $b$  they value the two increases from 0 to 0.25 and from 0.75 to 1 as greater as the decrease from 0.75 to 0.25 (which for facilitating the comparison may be split up into two imaginary decreases: one from 0.75 to 0.5 and one from 0.5 to 0.25). And they must do so

because of the concavity of the prioritarian value function. This preference for  $b$  is generated even with minimal degrees of priority.<sup>25</sup> - In studies conducted in 2002-2004 with 79 participants (students at the University of Siena and future teachers of philosophy in the district of Tuscany) who had to choose according to their moral intuitions between alternatives constructed in the fashion of  $a$  and  $b$  81.0% ( $n=64$ ) of the subjects preferred the analogue of  $b$ , i.e. decided in a prioritarian fashion, 13.9% ( $n=11$ )<sup>26</sup> preferred the analogue of  $a$ , and 1.3% ( $n=1$ ) found the analogues of  $a$  and  $b$  equivalent (3.8% ( $n=3$ ) gave no clear answer). This means, first, that the difference between  $a$  and  $b$  is not only technical gimcrackery but intuitively seen as making a practical difference and, second, there are more prioritarians around than is usually assumed.

Fleurbaey has seen prioritarianism only as a not well defined particular form of egalitarianism [Fleurbaey 2002, 1; 5-8]. But this opinion is due to his very vague and broad definitions of the two concepts. Broome instead defined 'prioritarianism' taking additive separability as its characteristic feature and 'egalitarianism' as those welfare ethics that fulfil the Pigou-Dalton condition and are not prioritarian [Broome 2002, 1 f.]. Problems with these definitions are that additive separability is an important but not the essential feature of prioritarianism and that the definition of 'egalitarianism' is only negative thus missing the characteristic feature of egalitarianism, too. In what has been developed here so far I hope to have provided strong and characterizing definitions of 'prioritarianism' and 'welfare egalitarianism', which distinguish them clearly and leave room for other forms of equity welfarism.

### 3. An Intuitive Determination of an Adequate Prioritarian Value Function

Material or criteriological ethics aims at practical application. It should provide ethical criteria apt to establish moral rankings between given options. What has been said about prioritarianism so far is not sufficient for doing so because it leaves room for indefinitely many prioritarian value functions, which will rank a given set of options very often in quite diverging ways. So exactly *one* adequate prioritarian value function has to be filtered out, which then can be used for practical application. In the present section this will be done by means of intuitions. In the following section then an internalist justification will be provided for the prioritarian value function chosen here.

The present section proceeds in two big steps. First, the general form of the prioritarian value function will be chosen by means of rather general and in parts only formal adequacy

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<sup>25</sup> Broome has developed two further examples for, probably, diverging rankings by prioritarianism and egalitarianism [Broome 1989a, 185; 2002, 3 f.]. These examples are illuminating but they are both based on further strong presuppositions.

<sup>26</sup> This number of the preferences for  $a$  as compared to that of those being indifferent seems rather high. It may be due to the fact that the sums in the examples used in the test because of errors of rounding were minimally higher for the analogue of  $a$  than for  $b$  so that utilitarians should have preferred  $a$  to  $b$ . The subjects preferring  $a$  to  $b$  so may have been utilitarians.

condition. This general form leaves open the degree of priority, i.e. if the valuation is nearer to utilitarianism (low degree of priority) or nearer to leximin (high degree of priority). The second step consists in calibrating the degree of priority relying on some prioritariums' intuitions about how to rank several options. One big advantage of this two-step procedure is that it separates more general and technical questions from questions of individual moral "taste". So it is open to several degrees of agreement with the final results, so that some people e.g. might accept the general form of the value function but prefer a different calibration.

### 3.1. Exponential Value Functions as the Most Adequate Group of Prioritarian Value Functions

We are then seeking a group of prioritarian value functions  $VP_\lambda$  differing in one parameter only, which may be called " $\lambda$ " and which expresses the degree of priority. For convenience the lowest degree of priority for the moment shall be fixed as being equal to 0, higher  $\lambda$  expressing stronger prioritarian inclinations. (Later on we will see that for some interesting groups of functions it is more natural to fix  $\lambda=1$  (instead of  $\lambda=0$ ) as the lower bound.) - It has already been emphasized that prioritarianism may be seen as a synthesis of utilitarianism and leximin. So it would be nice to have utilitarianism and leximin as the limiting cases of the group of value functions searched. This means it would be nice to have a group of functions with, first,  $\lambda=0$  being identical to utilitarianism, i.e.  $VP_0(u)=u$ , and, second, for that holds that for any set of options a sufficiently high  $\lambda$  creates a leximin ranking. This, let us call it the "*synthesis condition*", is not really *necessary*; there may be prioritarian value functions exactly expressing many people's intuitions without fulfilling the synthesis condition. But it is a desirable feature, and as long as we have the choice between otherwise equally apt groups of functions this feature makes the respective group preferable. - Between  $\lambda=0$  and the highest values for  $\lambda$  then should lie the real prioritarian value functions so that all functions of the group with  $\lambda>0$  must fulfil the defining conditions for prioritarian value functions, i.e. triple differentiability and  $VP'>0$ ,  $VP''<0$ , and  $VP'''>0$  throughout (cf. P4.1-3). - In addition, the real prioritarian functions should allow for moderate degrees of prioritarianism and must not all be extreme. So they should include moderate  $\lambda$  for that the prioritarian weight for  $u=0$  ( $VP'(0)$ ) is not immediately infinite and the weight for  $u=1$  not 0 ( $VP'(1)>0$ ). Conversely, the real prioritarian functions should not necessarily be bound to utilitarianism even in some part of their course in that they always include some more or less straight piece. Such straight pieces mean that the weight given to improvements remains (nearly) constant for some range of initial well-being. And this is contrary to the idea of a smooth decline of such weights. (Straight parts of the value function may come up to some form of need principle, where high priority is given to improvements for people under a certain well-being threshold and higher levels are supported to a lower degree according to utilitarian criteria. Even if such a type of valuation may be attractive for some people it, surely, is not prioritarian.)  $VP$  having no straight pieces technically means that the second derivation  $VP''$  remains below some negative value (e.g.  $VP''(u)<-0.2$ ). - Not a real necessity



but a technical convenience facilitating comparisons would be to normalize the functions for  $u$  as well as  $VP_\lambda(u)$  for some standard interval, which here is chosen as  $[0, 1]$  so that  $\langle 0, 0 \rangle$  as well as  $\langle 1, 1 \rangle$  are points of all functions of the group  $VP_\lambda$ . The standardization of the utilities however shall only comprise the vast majority of cases and not exclude rare and extreme utilities outside that interval, so that  $VP_\lambda$  should be defined also for utilities (a bit) outside  $[0, 1]$ . - A further but this time not only formal but material condition expressing some people's intuitions shall reflect the idea that prioritarian valuations consider *absolute* utility levels so that with increasing utility levels priority should diminish relatively. An operationalization of this idea is that proportional increases of complete utility distributions must not necessarily let the respective rankings unchanged. Consider e.g. the distributions  $a = \langle 0.1, 0.1 \rangle$  and  $b = \langle 0.3, 0 \rangle$ .  $b$  perhaps may be considered as being slightly better than  $a$  because an increase from a very low utility level of 0.1 to a good one like 0.3 outweighs the decrease from a very low utility (0.1) level to misery (0). (I do not want to say that prioritarians *must* value in this way; probably it is a limiting case. And if for some prioritarian this is not a limiting case for him there may be analogous limiting cases.) If we now redouble utilities, obtaining  $a^* = \langle 0.2, 0.2 \rangle$  and  $b^* = \langle 0.6, 0 \rangle$ , the argument may not hold any longer: the increase from 0.2, which is already a modest and no longer a very low level, to 0.6, a rather high level, does not outweigh the decrease from a modest level (0.2) to misery (0), so that we obtain the following rankings:

$VPT(a) = VPT\langle 0.1, 0.1 \rangle < VPT\langle 0.3, 0 \rangle = VPT(b)$ , but

$VPT(a^*) = VPT\langle 0.2, 0.2 \rangle > VPT\langle 0.6, 0 \rangle = VPT(b^*)$ .

This means, though the percental increase in  $u$  in going from  $a$  to  $b$  and from  $a^*$  to  $b^*$  (e.g. from 0.1 to 0.3 and from 0.2 to 0.6, respectively) in both cases is identical, the percental increase e.g. from  $VP(0.1)$  to  $VP(0.3)$  is higher than the percental increase from  $VP(0.2)$  to  $VP(0.6)$ . Such percental increases in  $VP(u)$  are diminishing with higher values of  $u$ . Technically this is expressed as: The elasticity of  $VP$  is decreasing.<sup>27</sup>

Summarizing, we have the following *adequacy conditions for groups of prioritarian value functions*:

*AP1: Group of value functions:* Prioritarian value functions  $VP_\lambda$  are elements of a continuous group of one-adic single-valued functions  $VP$  from the set of real numbers  $\mathbb{R}$  into the set of real numbers  $\mathbb{R}$  with the parameter  $\lambda$  and  $\lambda \geq 0$  for that the following conditions hold.

*AP2: Standardisation:*

1. All  $VP_\lambda \in VP$  are three times differentiable.
2. The interesting intervals are  $VP_\lambda: [0, 1] \rightarrow [0, 1]$ ; however the functions should behave well even a bit outside these intervals.
3. All  $VP_\lambda \in VP$  have the standardized points  $VP_\lambda(0)=0$  and  $VP_\lambda(1)=1$ .

*AP3: Prioritarian curvation:* For all  $VP_\lambda \in VP$  with  $\lambda > 0$  and for all  $u$  holds:

<sup>27</sup> The elasticity  $E$  of  $VP$  at the point  $u$  is defined as the percental change of  $VP(u)$  compared to the percental change of  $u$ .

$E(VP)(u) := [(VP(u+h)-VP(u))/VP(u)]/[h/u] = VP'(u) \cdot (u/VP(u))$ .

1.  $VP_\lambda'(u) > 0$ ,
2.  $VP_\lambda''(u) < 0$ ,
3.  $VP_\lambda'''(u) > 0$ .

*AP4: Synthesis of utilitarianism and leximin:*

1. Monotonous increase in  $\lambda$ : For all  $\lambda$  and all  $\delta > 0$  and all  $u$  with  $0 < u < 1$  holds:  $VP_\lambda(u) < VP_{\lambda+\delta}(u)$ .
2. Utilitarianism as lower bound:  $VP_0(u) = u$  and  $\lim_{\lambda \rightarrow 0} VP_\lambda(u) = u$ .
3. Leximin as upper extreme: For all  $n \in \mathbb{N}$  with  $n \geq 2$  and for all  $b, \delta$  with  $0 < \delta \leq b < 1 - \delta$  there is a  $\lambda_0 \geq 0$  so that for all  $\lambda_g$  with  $\lambda_g > \lambda_0$  holds:  $n \cdot VP_{\lambda_g}(b) > VP_{\lambda_g}(b-\delta) + n - 1$ . (The weakening of this condition for the case of only two persons is: For all  $b, \delta$  with  $0 < \delta \leq b < 1 - \delta$  there is a  $\lambda_0$  so that for all  $\lambda_g$  with  $\lambda_g > \lambda_0$  holds:  $2 \cdot VP_{\lambda_g}(b) > VP_{\lambda_g}(b-\delta) + 1$ .)
4. Moderate forms of prioritarianism included: There are moderate  $\lambda > 0$  with  $\lim_{x \rightarrow 0} VP_\lambda'(x) < \infty$  and  $VP_\lambda'(1) > 0$ .
5. No partial utilitarianism: There are (small)  $\lambda$  so that for all  $u$  with  $0 \leq u \leq 1$  holds:  $VP_\lambda''(u) < -0.2$ .

*AP5: Additional prioritarian feature: priority to absolutely small  $u$  (decreasing elasticity):* For all  $\lambda$  with  $\lambda > 0$  the elasticity of  $VP_\lambda(u)$  is decreasing with increasing  $u$  (i.e. for all  $u$  and  $\delta > 0$ :  $E(VP_\lambda)(u+\delta) < E(VP_\lambda)(u)$ ).

Philosophers and economists have proposed groups of prioritarian value functions that fulfil at least a big part of these adequacy conditions or single prioritarian value functions that can be generalized to such a group. Several times the square root has been used as an example for prioritarian value functions [e.g. Rabinowicz 2001; Klint Jensen 2002, 16]. Generalizing the basis of the root we obtain the group of:

*Root functions  $VP_{rr}$ :  $VP_{rr}(u) = u^{1/r}$ , with  $r \geq 1$  (cf. figure 6).*

Here as in the following I use the same letter, e.g. " $r$ ", for abbreviating the group of functions, i.e. the type of curves, as well as the parameter  $\lambda$ .  $\lambda$  itself is not used here because the respective parameters for the different groups do not mean the same and assume quite different values. The root value functions are already one of the announced cases in which it is more convenient to let the parameter  $r$  (or  $\lambda$ ) begin with 1 instead of 0. (But of course this does not change anything. One could have defined the group as  $VP_{rr}(u) = u^{1/(r+1)}$  and then let  $r$  begin with 0.) - A further group of functions, the *iso-elastic functions*, and a variant of them, which I call "*Atkinson functions*", has been proposed and used by many economists:<sup>28</sup>

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<sup>28</sup> Sen in his book "On Economic Inequality" already used the function  $VT = 1/i \cdot u^i$ , with  $i \leq 1$  [Sen 1973, 21], which is equivalent to the iso-elastic function. But he used it only by the way in an example and only for a two person case. - The Atkinson function has been proposed by Atkinson / Stiglitz [1980, 340; 402], without quoting any source and without motivating this particular form. But the proposal may have been inspired by Atkinson's earlier use of the iso-elastic function as the utility function over income in a utilitarian context [Atkinson 1970, 251; 257; 1975, 48]. Even in the later book [Atkinson / Stiglitz 1980] the authors oscillate between interpreting the Atkinson function as a utility function over income in a utilitarian framework and a function expressing inequality aversion (sic!) [cf. Atkinson / Stiglitz 1980, 404]. - The iso-elastic function later has been used e.g. by: Boadway and Bruce [1984], Drèze and Stern [1987, 959 f.], Wagstaff [1991, 35], Fankhauser, Tol and Pearce [1997, 257].

*Iso-elastic functions*  $VP_{ii}$ :  $VP_{ii}(u) = (1/(1-i)) \cdot u^{1-i}$ , for  $0 \leq i < 1$  and for  $i > 1$  (cf. figure 7).

*Atkinson functions*  $VP_{aa}$ :  $VP_{aa}(u) = (1/(1-a)) \cdot (u^{1-a} - 1)$ , for  $0 \leq a < 1$  and for  $a > 1$ ;  $VP_{a1}(u) = \ln(u)$  (cf. figure 8).

$\ln(u)$  is the left and right limes of  $VP_{aa}(u)$  for  $a \rightarrow 1$ . - A group of exponential functions has been proposed by the author:<sup>29</sup>

*Exponential functions*  $VP_{ee}$ :  $VP_{ee}(u) = (e/(e-1)) \cdot (1-e^{-u})$ , for  $e > 1$ ;  $VP_{e1}(u) = u$  (cf. figure 9).

(Here "e" is a parameter and does not mean Euler's number.)  $VP_{e1}(u) = u$  is the right limes of  $VP_{ee}(u)$  for  $e \rightarrow 1$ . - Some further groups of functions *prima facie* suitable are these:

*Power functions*  $VP_{pp}$ :  $VP_{pp}(u) = 1 - (1-u)^p$ , with  $p \geq 1$  (cf. figure 10).

*Hyperbolic functions*  $VP_{hh}$ :

$$VP_{hh}(u) = \frac{h^2}{h^2-1} - \frac{h/(h^2-1)}{((h^2-1)/h) \cdot u + 1/h}, \text{ for } h > 1; VP_{h1}(u) = u, \text{ for } h = 1 \text{ (cf. figure 11).}$$

Finally one may add what at first sight seems to be the straightforward way of representing a compromise of utilitarianism and leximin, namely taking one of the just considered functions with a high  $\lambda$ , e.g.  $VP_{e1000000}$  and mix it in different concentrations with utilitarianism:

*Mixture functions*  $VP_{mm}$ :  $VP_{mm}(u) = m \cdot VP_{e1000000}(u) + (1-m) \cdot u$ , with  $0 \leq m \leq 1$  (cf. figure 12).

Which of these groups of functions fulfils the adequacy conditions developed above?

The group of the *root functions*  $VT_{rr}$  does not approach sufficiently to leximin for high  $r$ , and its slope at the point  $u=0$  even for low  $r$  is already infinite ( $VP_{rr}'(0)=\infty$ ). So it is not suited to represent a compromise of utilitarianism and leximin. In addition it is iso-elastic, i.e. has the same elasticity for all  $u$ . Summing up, the group of root functions violates AP4.3, AP4.4 and AP5.

The group of *power functions* even for small  $\lambda$  has slopes in  $u=0$  approaching to infinity ( $\lim_{u \rightarrow 0} VP_{pp}'(u) = \infty$ ), thus violating AP4.4. In addition the functions do not behave well for  $u$  outside the interval  $[1, 0]$ .

The group of *hyperbolic functions* even with very high  $\lambda$  does not always lead to leximin rankings, thus violating AP4.3. Apart from that hyperbolic functions express prioritarian intuitions quite well.

Broome states that when he wrote his book "Weighing Goods" (published in 1991) "prioritarianism was well established amongst economists, but was only just being discovered by philosophers" [Broome 2002, 1]. These are rather strong assertions. The authors just cited all use a prioritarian welfare function. However the ideas behind this usage were not clearly prioritarian (independent of not using the word "priority") but often egalitarian and at best sought to find a middle way in between utilitarianism and leximin. The idea of priority, i.e. giving greater weight to improvements for those worse off, is missing in any case. But we have to keep in mind that Broome uses "prioritarianism" in a weaker sense (additive separability plus Pigou-Dalton condition) as is done here. Among philosophers, on the other hand, obviously prioritarian ideas can be found already in Nagel's Tanner Lecture from 1977 [Nagel 1978]. Nagel seeks to find a middle way between utilitarianism and leximin, too, but he expresses the idea of priority to the worse off. However even his discussion is still rather crude, and he confuses priority and equality, too.

29

I have developed this function as well as its internalist justification (sketched in the following section) in my habilitation thesis submitted in 1992. But this thesis has been published only in 2000 [Lumer 2000] and only in German. (Section 7.2 (pp. 589-616) contains the internalist justification, section 7.3 (pp. 616-632) the development of the exponential value function.) A short condensation of this material is: Lumer 1997.

Fig. 6a: Root functions  $VP_{rr}$ :  $VP_{r1}$ ,  $VP_{r2}$ ,  $VP_{r4}$ ,  $VP_{r10}$

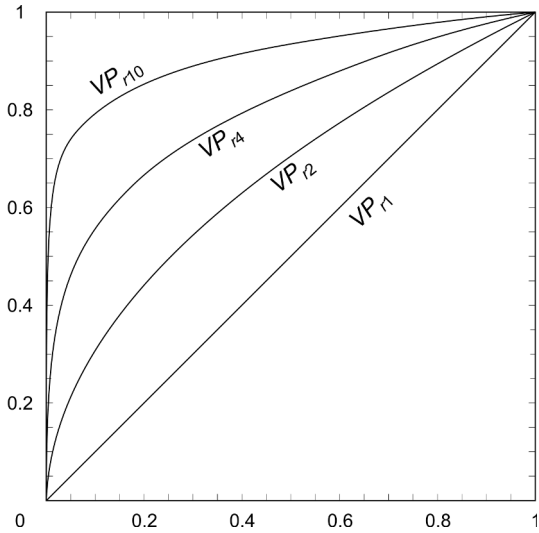


Fig. 6b: First derivation  $VP_{rr}'$  of root functions

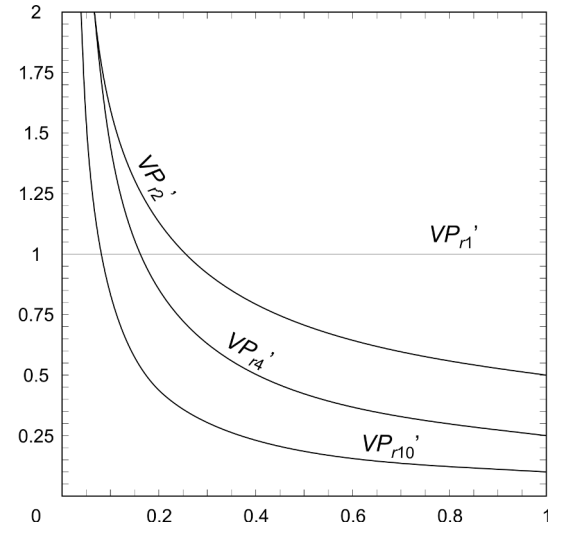


Fig. 7a: Iso-elastic functions  $VP_{ii}$ :  $VP_{i0}$ ,  $VP_{i0.5}$ ,  $VP_{i0.9}$ ,  $VP_{i1.5}$ ,  $VP_{i2}$

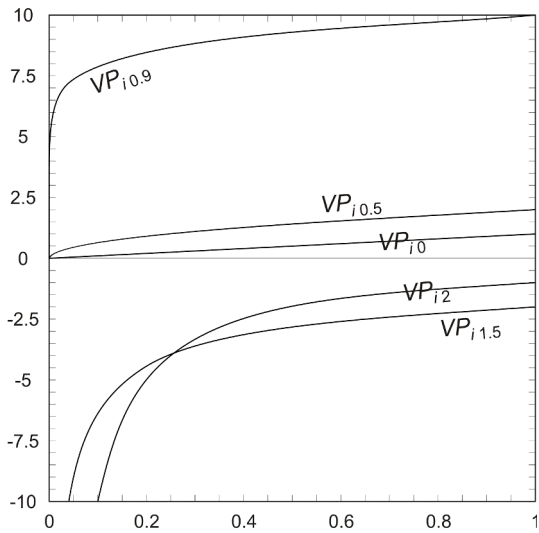


Fig. 7b: First derivations  $VP_{ii}'$  of iso-elastic functions

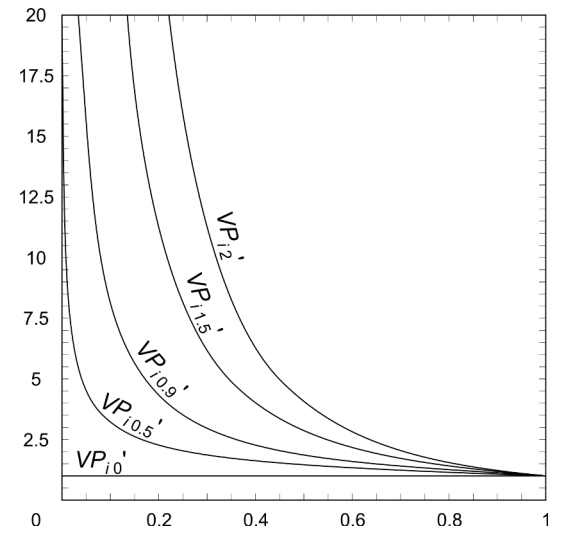


Fig. 8a: Atkinson functions:  $VP_{a0}$ ,  $VP_{a0.5}$ ,  $VP_{a1}$ ,  $VP_{a1.5}$ ,  $VP_{a2}$

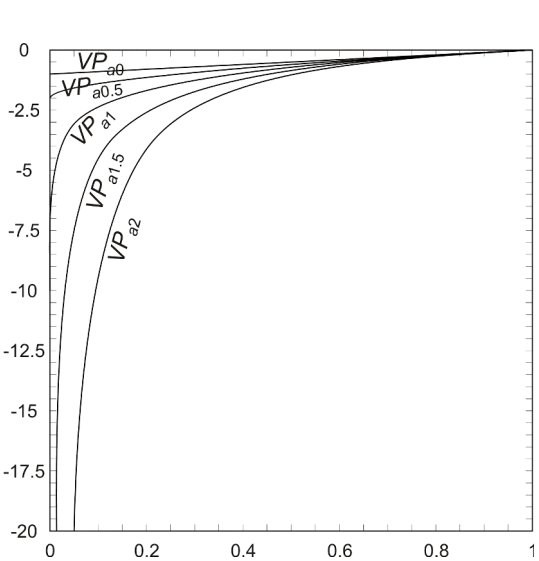


Fig. 8b: First derivations  $VP_{aa}'$  of Atkinson functions

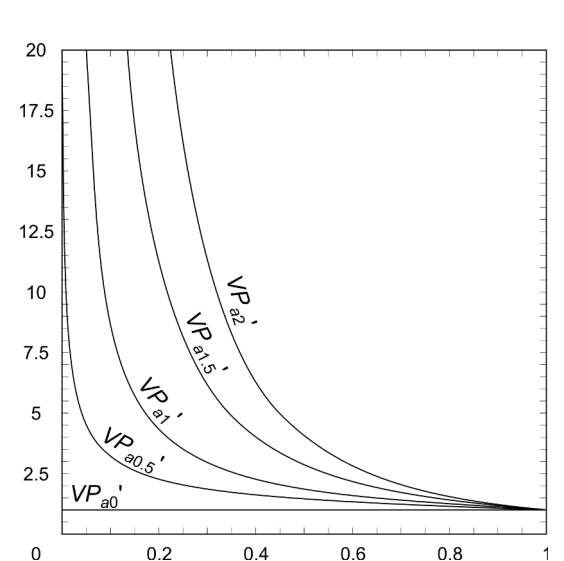


Fig. 9a: Exponential functions:  $VP_{e1}$ ,  $VP_{e7}$ ,  $VP_{e19}$ ,  $VP_{e500}$

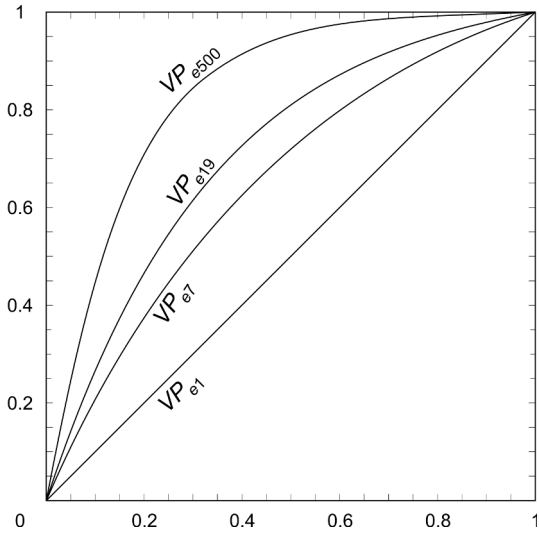


Fig. 9b: First derivations  $VP_{ee}'$  of exponential functions

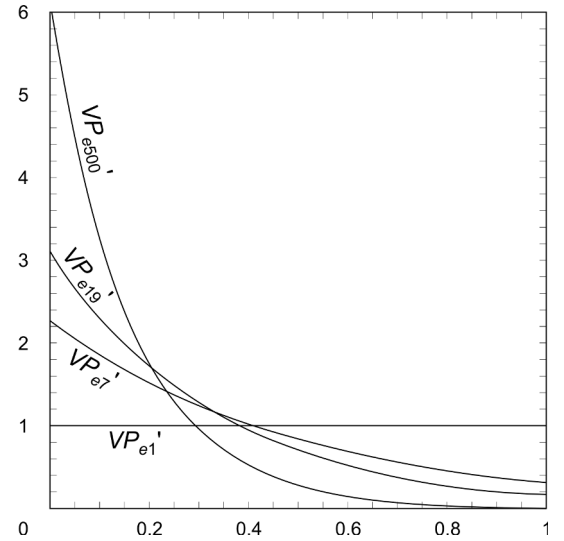


Fig. 10a: Power functions:  $VP_{p1}$ ,  $VP_{p1.5}$ ,  $VP_{p2}$ ,  $VP_{p4}$ ,  $VP_{p10}$

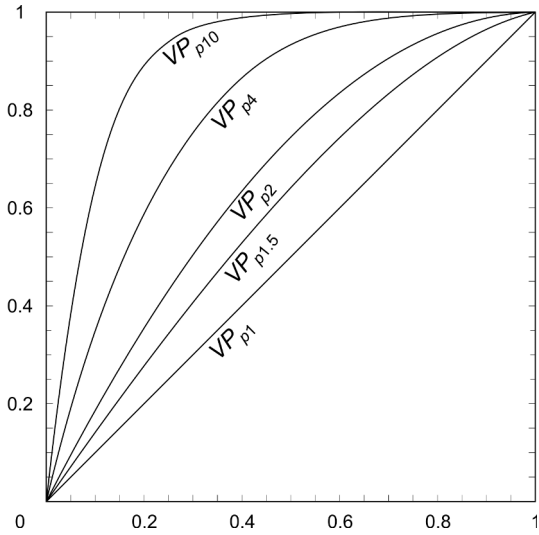


Fig. 10b: First derivation  $VP_{pp}'$  of power functions

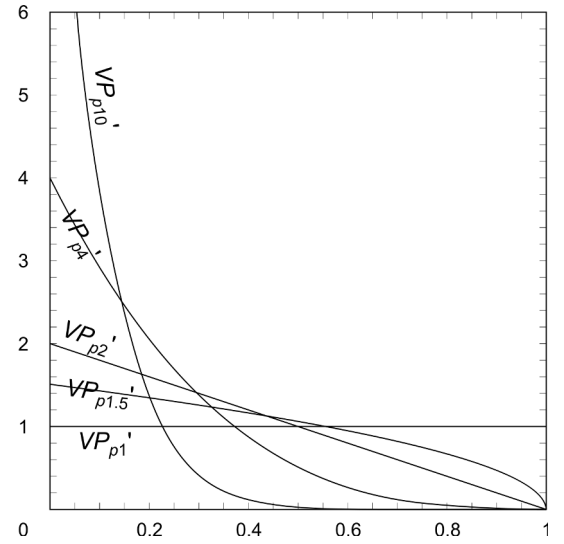


Fig. 11a: Hyperbolic functions:  $VP_{h1}$ ,  $VP_{h2}$ ,  $VP_{h4}(u)$ ,  $VP_{h10}$

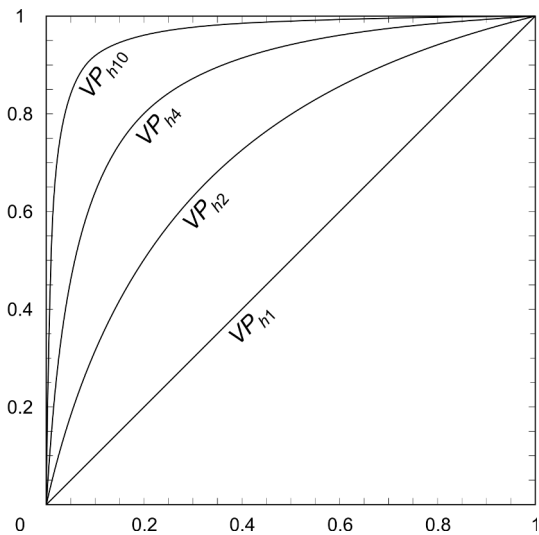


Fig. 11b: First derivations  $VP_{hh}'$  of hyperbolic functions

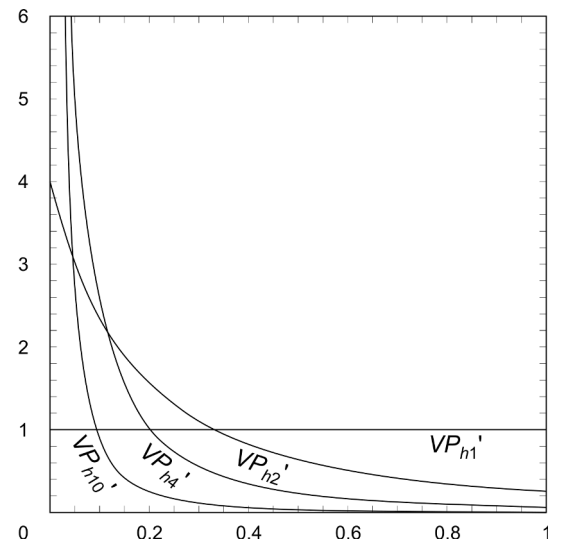
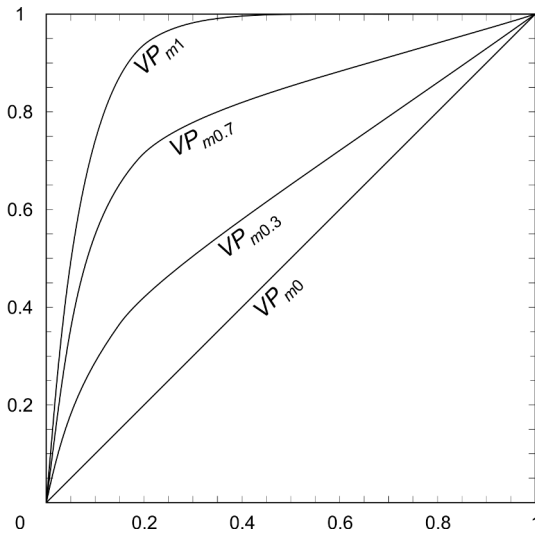
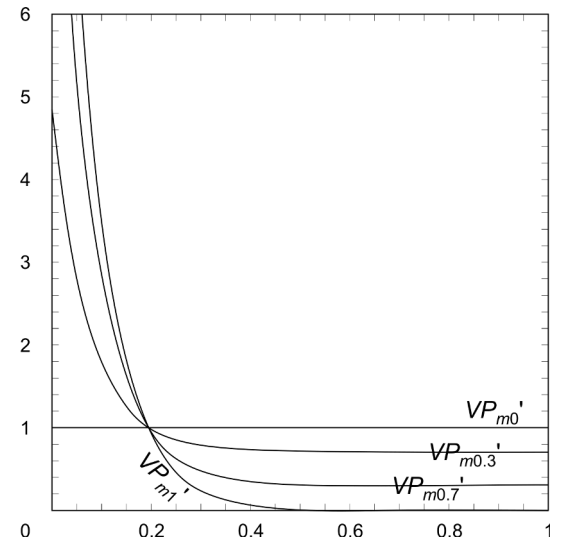


Fig. 12a: Mixture functions:  $VP_{m0}$ ,  $VP_{m0.3}$ ,  $VP_{m0.7}$ ,  $VP_{m1}$ Fig. 12b: First derivation  $VP_{mm}'$  of mixture functions

The group of *mixture functions* does rather badly. First, having chosen one of the other groups of function, e.g.  $VP_{ee}$ , for representing the leximin extreme there is not *the*  $\lambda$  (for  $VP_{ee}$ : not *the*  $e$ ) above which every set of options will be ranked in leximin fashion. (Only the reverse holds: For every set of options there is some  $\lambda_0$  above which leximin rankings will be created.) Hence having once fixed the high  $e$  then there may be extreme sets of options that are not ordered as by leximin. So the mixture functions, strictly speaking violate AP4.3. Second and much more important, leximin functions for  $u$  a bit above 0 approach to a straight line (running from  $\langle 0, 1 \rangle$  to  $\langle 1, 1 \rangle$ ); mixing this with the linear utilitarianism leads to a nearly straight upper part of the function, i.e. some partial utilitarianism, thus violating AP4.5.

The group of *iso-elastic functions* actually consists of two groups of functions, the first for  $0 \leq i < 1$  and the second for  $i > 1$ . The first group with the exponent  $1-i$  now ranging between 0 and 1 is a group of root functions. If it is positive-linearly transformed in the interval  $[0, 1]$ , i.e. letting away the initial factor  $1/(1-i)$ , we simply obtain the root functions discussed above ( $\lambda$  is defined differently now, though) - with all their defects. The second group with  $i > 1$  now converts into a group with negative exponents. Trying to shift these functions by positive-linear transformation in the interval  $[0, 1]$  turns out to be difficult because the value of  $u=0$  is  $-\infty$  (for  $i > 1$ :  $VP_{ii}(0)=-\infty$ ). That the function fits the interval  $[0, 1]$  is only convenient, though, and no substantial requirement. In addition, however, the limes of the first derivation for  $u$  approaching 0 is  $+\infty$  ( $\lim_{u \rightarrow 0} VP_{ii}'(u) = +\infty$ ) - in contrast to AP4.4. A further problem is that, as indicated in their name, iso-elastic functions have a constant elasticity, which is identical to their exponent, i.e.  $1-i$ . So they violate AP5, too.

*Atkinson's functions*, being very similar to iso-elastic functions share most of their advantages and disadvantages. One of their advantage is that the resulting curves for different  $a$  change continuously around  $a=1$ . Because of the subtraction of 1 (or more precisely: subtraction of  $1/(1-a)$ ) the functions are no longer iso-elastic. But their elasticity now behaves rather irregularly, e.g. for  $a=2$ , it first decreases from  $E(VP_{a2})(0)=-1$  then jumps up to  $E(VP_{a2})(1)=\infty$ . This is in contrast to AP5, too.

*Exponential functions* are the only group of functions among those scrutinized here that fulfil all adequacy conditions introduced above. This can be seen in the most part already from the respective graphs. The fact that they have leximin as their upper extreme (AP4.3) cannot be seen in this way; but I have proved that elsewhere [Lumer 2000, 628]. An example for their giving priority to absolutely small levels of well-being (AP5) is that they may reverse rankings as a consequence of the proportional increase of the well-being distribution. Taking the examples given above and assuming  $e=19$  one obtains:

$$\begin{aligned} VPT_{e19}(a) &= VPT_{e19}\langle 0.1, 0.1 \rangle = 0.269+0.269 = 0.538 < 0.619 = 0.619+0 = VPT_{e19}\langle 0.3, 0 \rangle = \\ &VPT_{e19}(b), \\ VPT_{e19}(a^*) &= VPT_{e19}\langle 0.2, 0.2 \rangle = 0.470+0.470 = 0.940 > 0.875 = 0.875+0 = VPT_{e19}\langle 0.6, 0 \rangle = \\ &VPT_{e19}(b^*). \end{aligned}$$

### 3.2. Calibrating the Prioritarian Value Function: Utilex

Because of these merits the group of exponential value functions  $VP_{ee}$  in the following will be adopted as the most adequate group of prioritarian value functions. What remains to be done in this subsection is to calibrate this function, i.e. to determine some intuitively adequate value for the parameter  $e$ .

The (positive-linear transformations of the) following tables have been presented to some non-philosophers among my acquaintances.

*Table 1: Well-being of different groups of persons in various social orders:*

$u_{ij}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$
$s_1$	0	0	0.2	0.2	0.2	0.5	0.45	0
$s_2$	0.4	0.6	0.4	0.6	0.6	0.5	0.45	0.4
$s_3$	0.6	0.6	0.6	0.5	0.6	0.5	0.45	0.6
$s_4$	1	0.6	0.8	0.5	0.6	0.5	0.45	0.94
$\Sigma$	2.0	1.8	2.0	1.8	2.0	2.0	1.8	1.94

*Table 2: Well-being of different groups of persons in various social orders:*

$u_{ij}$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$
$s_1$	0.1	0.2	0.2	0.2
$s_2$	0.2	0.2	0.3	0.3
$s_3$	0.8	0.8	0.5	0.6
$s_4$	0.9	0.6	1	0.8
$\Sigma$	2.0	1.8	2.0	1.9

It was explained to them that the  $s_i$  were individuals or homogenous groups of equal size, that the  $a_j$  were different types of social orders, which would bring about a well-being of the indicated degree for the individuals and I explained them the meaning of utility levels 0 and 1 as done at the beginning of this paper. Then I asked them to rank these options according to their moral intuitions,

first alternatives  $a_1$  to  $a_8$  then  $a_9$  to  $a_{12}$ . The three most important intuitive rankings are represented in table 3 together with some important theoretical rankings.

*Table 3: Intuitive and theoretical moral rankings of options from tables 1 and 2:*

intuition <sub>I1</sub> [=VPT <sub>e19</sub> =VPT <sub>e20</sub> ]:	$a_6 > a_7 > a_5 > a_3 > a_4 > a_2 > a_1 > a_8.$	$a_{12} > a_{11} > a_{10} > a_9.$
intuition <sub>I2</sub> :	$a_6 > a_7 > a_5 > a_3 > a_4 > a_2 > a_1 > a_8.$	$a_{11} > a_{12} > a_{10} > a_9.$
intuition <sub>I3</sub> [=VPT <sub>e7</sub> ]:	$a_6 > a_5 > a_3 > a_7 > a_4 > a_1 > a_8 > a_2.$	$a_{11} > a_{12} > a_{10} > a_9.$
utilitarianism [=VPT <sub>e1</sub> ]:	$a_1 = a_3 = a_5 = a_6 > a_8 > a_2 = a_4 = a_7.$	$a_9 = a_{11} > a_{12} > a_{10}.$
leximin [=VPT <sub>e≥500</sub> ]:	$a_6 > a_7 > a_5 > a_4 > a_3 > a_2 > a_1 > a_8.$	$a_{12} > a_{11} > a_{10} > a_9.$
VPT <sub>e10</sub> :	$a_6 > a_5 > a_7 > a_3 > a_4 > a_1 > a_2 > a_8.$	$a_{12} > a_{11} > a_{10} > a_9.$

So the most important intuitions were not utilitarian, for the participating subjects the distribution of well-being was not irrelevant. They all ranked  $a_9$  behind  $a_{10}$ , and  $a_1$  behind  $a_7$  though the total well-being in  $a_{10}$  and  $a_7$  is higher. Improvements for the worse off seem to have been valued higher as those for the better off. A drastic case is the intuitive preference for  $a_{10}$  over  $a_9$ .  $s_2$  and  $s_3$  remain unchanged, but for improving the well-being of  $s_1$  from 0.1 to 0.2 the subjects were willing to accept the much bigger deterioration for  $s_4$  from 0.9 to 0.6. And the subjects did not rank in leximin fashion either. So in all intuitive rankings  $a_3$  was preferred to  $a_4$ . By going from  $a_3$  to  $a_4$  one could increase the well-being of the second worst off from 0.4 to 0.5 but only for the price of reducing the well-being of the best off from 0.8 to 0.5. Leximin would do that but not the subjects asked.

Two of the three most important intuitions can be represented by the exponential value function, namely intuition *I1* by VPT<sub>e19</sub> and VPT<sub>e20</sub>, and intuition *I3* by VPT<sub>e7</sub>. Intuition *I2* cannot be represented by the exponential value function because the first group of options ( $a_1$ - $a_8$ ) is ranked as in intuition *I1*, representable by VPT<sub>e19</sub> and VPT<sub>e20</sub>, whereas the second group ( $a_9$ - $a_{12}$ ) is ranked as in intuition *I3*, representable by VPT<sub>e7</sub>.

The interviewed sample was rather small, and many people will rank the options quite differently. However, even this small experiment, first, has proved that at least some people's intuitions can be represented by some exponential value function. Second, for the present task of calibrating the exponential value function only intuitions of prioritarrians are of importance. And the results obtained confirm that VP<sub>e7</sub> up to VP<sub>e20</sub> have the correct order of magnitude. A further unanimous (among prioritarrians) reduction of this range on an intuitive basis will not be possible. So on the intuitive basis we have to leave it at that. However in the next section an internalist justification for VPT<sub>e19</sub> will be given, so that VPT<sub>e19</sub> here will be proposed as the most adequate prioritarian value function. The graphs of the value function VP<sub>e19</sub> and its first derivation are represented in figure 9, some values of these functions are listed in table 4. That the slope of VP<sub>e19</sub> for  $u=0$  is 3.108 (VP<sub>e19</sub>'(0)=3.108) and the slope for  $u=1$  is 0.164 (VP<sub>e19</sub>'(1)=0.164) means: Improvements for somebody with a utility level of 0 are valued 18.95 (= 3.108/0.164) times higher than improvements for someone with the very high utility level of 1.



Table 4: Some values of the prioritarian value function  $VP_{e19}$  (=utilix)

$u$ :	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$VP_{e19}(u)$ :	0.000	0.269	0.470	0.619	0.730	0.813	0.875	0.921	0.955	0.981	1.000
$VP_{e19}'(u)$ :	3.108	2.315	1.725	1.285	0.957	0.713	0.531	0.396	0.295	0.220	0.164

I have named  $VPT_{e19}$  "utilix", which is a blend of "utilitarianism" and "leximin", thus expressing that it is a synthesis of these two welfare functions. For practical applications a further standardization is necessary, which establishes exactly what values of  $VP_{e19}$  refer to what level of well-being. This has been done elsewhere [Lumer 2002a, 26-32; 65-69; in particular 68]<sup>30</sup>, and the result has been applied to morally value practical options (with respect to climate change) [ibid. 65-85]. So the practicability and usefulness of utilix has been proved.

## 4. An Internalist Justification of Prioritarianism in General and Utilix in Particular

### 4.1. An Internalist Conception of Justifying Morals

Prioritarianism by most of its defenders has been adopted for intuitive reasons only - at least publicly they provide not more than systematisations of their prioritarian intuitions. But some theorists have developed a theoretical justification of prioritarianism, based on risk aversion in the original position [cf. e.g. Atkinson / Stiglitz 1980, 340; Hurley 1989, 368-382]. This justification is based on the Rawlsian [Rawls 1958; 1971], Harsanyiian [Harsanyi 1955; 1977a, 48-83; 1977b] framework of an impartial decision, operationalized via a hypothetical ignorance of one's proper qualities and position in society so that the decider may end up in everybody's skin. Harsanyi has argued that in such a situation one should consider the various possibilities as equiprobable and treat them risk-neutrally; this then leads to utilitarianism. Rawls has argued that in such a situation of uncertainty the maximin criterion for rational decisions should be applied, which then leads to a moral maximin criterion too. Prioritarians have criticized both justifications: Maximin (or leximin) is equivalent to infinite risk aversion, which seems to be nearer to paranoia than rationality, and Harsanyi's risk-neutrality is not rational either. The right maxim in such situations is moderate risk aversion, which together with the framework's assumptions leads to prioritarianism as the impartial and morally just welfare function.

But this justification is problematic. First, justifying a particular moral value function relying on attitudes towards risk means to rely on a much too technical feature that has little to do with moral questions. At least intuitively it is surprising that such an important moral question like

<sup>30</sup> The standardisation of utilities is such that  $u=1$  corresponds to a life of 186.4 years length and of the social mean well-being in present industrial societies (or to a life of 93.2 years with an individual mean well-being twice as high as the social mean well-being etc.). So a utility of 1 is a very rare case.

the decision between prioritarianism and e.g. utilitarianism should be taken for technical reasons. There is a discrepancy here, which shows up even in a very concrete way: Of course, there are prioritaricians who are not moderately risk averse but e.g. risk neutral or even risk seeking; and there are moderately risk averse people who are not prioritaricians. Second, and still more important, relying on the original position framework means to argue on the basis of very strong premises, which are accepted only by a rather small group. In addition, as Rawls has emphasized, his method is intuitionistic. The whole construction of the initial position from the very beginning is only a way of systemizing moral intuitions; it is not intended to provide any deeper justification.

If systematisations of moral intuitions are not real justifications of moral criteria what else are such justifications?<sup>31</sup> The formal point of departure for a conception of (fundamental) moral justification is that such justification consists in a valid and sound argument establishing the thesis that the object in question - in this case a moral criterion - has certain qualities; this thesis may be called the "*justifying thesis*" and the relevant quality the "*justifying quality*". The crucial question then is: What is the justifying quality for moral criteria? There are three adequacy conditions for such justifying qualities:

*AJ1: motivational impact:* The justifying thesis is motivating in the sense that a prudent person who is convinced of this thesis adopts the moral criterion practically. And this means that if this person in addition is convinced that a certain norm or action fulfils this criterion then the person is motivated, at least slightly, to follow the norm and to realize that action. The degree of motivation has to be sufficient for realizing, perhaps together with other motives and only historically in the long run, the norm and such actions. - This is the *practical* component of the conception of justification. It shall guarantee that moral justification does not provide whatever insights about the moral in question but practically relevant insights leading to the realization of that moral.

*AJ2: prudential acceptability:* The motivational impact of the conviction rests on motives (or general practical preferences) that are apt as a basis for a prudential desirability function. Some important conditions for such aptitude are stability of the motivation in the face of new and true information and temporal stability of the motivation. - Prudential acceptability is the *rational* component of the conception of justification. It guarantees that the respective motivation has not been accepted for naivety and disinformation only, and that it is stable over time and so permitting long-term planning.

*AJ3: moral instrumentality:* 1. The actions and norms selected by the criterion in question as being moral must have those formal qualities that by all developed ethics are considered to be morally relevant. 2. There is at least one developed moral already practically adopted by some people that roughly takes the same actions and norms to be moral as the justified moral criterion. 3. And the whole conception of morals to be justified has to fulfil the function that is implicitly assumed in developed ethics. With respect to socially binding morals the function of moral value systems is *consensualistic* in a specific meaning: The sense of a socially binding value system is to provide an

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<sup>31</sup> The conception of a fundamental moral justification sketched in the following has been developed and defended in: Lumer 2000, 30-46; Lumer 1999.

interpersonal uniform and binding order of values and valuation, which then shall resolve interpersonal conflicts of interests. - This is the *moral* component of the conception of moral justification. Without this component one could not say that the actions, norms and valuations established by the conception of justification have been *morally* justified.

Conditions AJ1 and AJ2 in the end imply that moral value functions must be some part of the individuals' prudential desirability function. And condition AJ3.3 adds to this that this part of the prudential desirability function must be interpersonally (more or less) identical, i.e. the same objects must be valued in the same way. This means it is not sufficient that the valuations are interpersonally *analogous* in that they refer to analogous objects, e.g. as in 'I detest *my* pain as you detest *yours*'; they must refer to the same objects, e.g. as in 'I detest *your* pain as you detest *your* pain and as anybody else does'. An empirical inquiry of many respective candidates has revealed that sympathy (of which compassion is the negative half) is the most important motive that may lead to such interpersonally identical valuation [cf. Lumer 2002b], or more precisely: it is a certain form of sympathy, a sort of anonymous or universal sympathy with people not well known. And it is not by chance that sympathy in the history of ethics has played an important role as a foundational motive (e.g. in Schopenhauer). Though sympathy is only a rather weak motive its interpersonal identity makes it apt to form the basis of a socially common value function, which in cases of social conflicts hopefully may tip the scales. On this mechanism a whole strategy of realizing morals historically in the long run can be based [cf. Lumer 2002a, 93-95]. So in the following a modern form of ethics based on sympathy will be developed.

## 4.2. Justifying Utilex Based on Sympathy <sup>32</sup>

Now one may share this particular internalist approach to moral justification or not. However, even for different reasons and approaches it may be interesting to scrutinize what kind of moral value function results from an ethics based on sympathy. The main task of the following justification of a particular moral value function is to develop a - simplifying - mathematical model of how the well-being of other persons whom we neither like nor dislike in a particular manner is reflected in our expected sympathy, i.e. the expected amount of our feelings of positive and negative sympathy, i.e. of sharing positive and negative feelings of these people. So, briefly, the model informs about the extent of our sympathy depending on other persons' well-being. The most important *simplifying assumptions* of this model are: 1. The object of our sympathy is the assumed well-being of our objects of sympathy. 2. Errors in our assumptions about other persons' well-being statistically adjust each other. 3. The model deals with universal sympathy only, i.e. a kind of sympathy we feel for strangers whom we neither like nor dislike in a particular way and whose behaviour we do not judge morally. 4. In a very flexible society like ours the chances to be confronted with the lot of other people are equal for all objects of sympathy. And the salience of the fate of other people statistically is equally distributed. 5. The intensity of our compassion depends

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For references to a more extensive exposition of the following justification see above, fn 29.

on the intensity and duration of considering it. But again the expected values of these two quantities are interpersonally equal for all objects of sympathy. 6. Prudent persons *have* feelings of sympathy and do not try to avoid them.

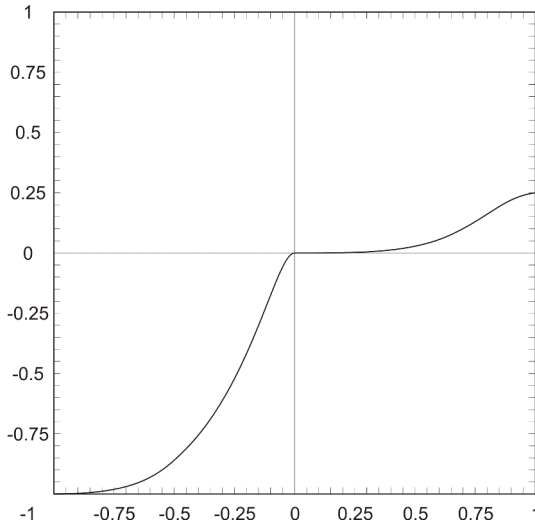
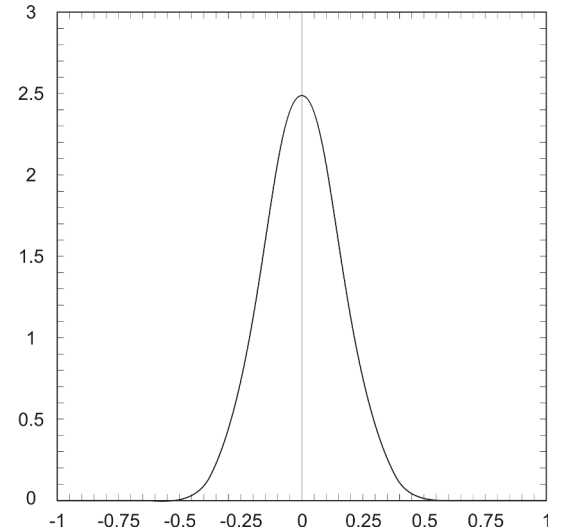
The first step in developing this model is to determine the intensity of our sympathy depending on the assumed condition of the object. Consider figure 13. The  $x$ -axis of this figure represents the object's well-being; positive values represent pleasant feelings, negative values represent unpleasant feelings. The well-being represented in this curve is the more or less *momentary* well-being according to the subject's (i.e. the moral agent's) present assumptions. So it is not the welfare of a whole life, to which the moral value functions considered so far referred and which was represented by the  $x$ -axes of the figures above. Momentary well-being here will be represented by the variable " $x$ ", whereas the welfare of a whole life, i.e. the integral of  $x$  over the whole life, is represented as before by the variable " $u$ ". The  $y$ -axis of figure 13 represents the appertaining sympathy, negative values representing pity, and positive values representing pleasant feelings of sharing joy. The other person's well-being as well as the sympathy is normalized in the interval  $[-1, 1]$  with 0 being the point of indifference (or non-feeling) of the other person's well-being as well as of the subject's sympathy. The variable for the resulting momentary sympathy is " $c$ " (for "compassion" - " $s$ " for "sympathy" would be better but has been used already for "subject"). Plausible assumptions about the function  $S$  from momentary well-being to sympathy are the following. The sympathy function  $S$  increases monotonously. On a neutral well-being we react sympathetically indifferent; i.e. the function includes the point  $\langle 0, 0 \rangle$ . Negative sympathy, i.e. pity, is much more intensive than positive sympathy. As I could ascertain in some interviews testing the willingness to exchange packages of such feelings with different durations, pity with the most extreme sort of suffering was 4 to 10 times more intensive than positive sympathy with the most extreme form of joy. Conservatively I have taken 4 to be the right relation. The most extreme points of the function  $S$  then are  $\langle -1, 1 \rangle$  and  $\langle 1, 0.25 \rangle$ . Empirically our normal well-being most of the time is not very extreme, ranging between 0 and 0.4. Our sympathetic reaction to this kind of normal well-being is minimal. Outside of this region of normalcy sympathy's intensity is increasing rapidly, though much more rapidly in the negative than in the positive direction. When approaching to extreme states of well-being sympathy will be satiated, i.e. approaching only smoothly to its highest degrees. - From these assumptions results the sympathy function of figure 13 with the following mathematical formula ( $s$  = subject;  $o$  = object;  $S_{s,p}$  = the degree of  $s$ ' sympathy because of  $p$ ;  $WBo,t = o$ 's well-being at time  $t$ ).

$$S_{s,(WBo,t=x)} = S(x) =$$

$$1. \text{ for } -1 \leq x \leq 0 : -0.15 \cdot (x+1)^{23} + 1.15 \cdot (x+1)^3 - 1;$$

$$2. \text{ for } 0 \leq x \leq 1 : -0.25 \cdot x^8 + 0.5 \cdot x^4.$$

The most important feature of this function is its non-linearity: Pity is much more intense than positive sympathy; and normal states of well-being (between 0 and 0.4) are nearly neglected by our sympathy.

Fig. 13: Sympathy  $S(x)$  depending on assumed well-being  $x$  of the object of sympathyFig. 14: Distribution  $PD(x)$  of well-being  $x$  for  $x_\mu=0$ 

The second step of the model is to find out the *intrapersonal distribution* of various well-being states for different objects of sympathy. In order to establish the extent of sympathy we need not know the exact course of the object's momentary well-being over time but only the proportional duration of the single levels of well-being during the whole life. Again simplifying, I assume that these levels of well-being are distributed normally, as represented in figure 14. This curve  $PD$  means that the well-being values under the top of the curve are realized most frequently, and better and worse values of well-being correspondingly more rarely. The open parameters of a normal distribution are 1. the mean  $x_\mu$  and 2. the standard deviation  $x_\sigma$ , i.e. 1. which level of well-being empirically is the mean level and happens to occur most frequently or, expressed graphically: above which value for  $x$  does the middle and top of the curve lie?; 2. how often do other levels of well-being occur or - again expressed graphically - how steep or flat is the curve? Empirical research on well-being has revealed that the interpersonally most extreme long-term means of well-being of the unhappiest and the happiest people, positively-linearly transformed in our scale  $[-1, 1]$ , lie between 0 and 0.4 ( $0 \leq x_\mu \leq 0.4$ ), so that the happiest people in the long run arrive at a mean of 0.4. Again simplifying, I assume that the means of well-being of happy and unhappy people are interpersonally different whereas their standard deviations are identical. Relying on some plausible assumptions about the absolute duration of very extreme feelings the standard deviation can be calculated as being equal to  $x_\sigma=0.16$ . With these assumptions one gets a *bundle* of curves of normal distributions, each representing the distribution of well-being levels for a range of typical, more or less happy individuals. All these curves are equally shaped but their means range - according to the general individual happiness - from 0 to 0.4; i.e. the curves are shifted to the left or to the right.

$$PD(x) = 1/(0.16 \cdot \sqrt{2\pi}) \cdot e^{[-(x-x_\mu)^2/(2 \cdot 0.16^2)]}, \quad 0 \leq x_\mu \leq 0.4.$$

The third step is to multiply the probabilities given by the normal distribution of well-being ( $PD(WBo, t=x)$ ) with the sympathy function  $S(x)$  and to calculate the integral from -1 to 1 over this product function. The result of this operation is the extent of sympathy  $ES(x_\mu)$ , i.e. the integral or 'balance' of all feelings of sympathy which one expects to feel for a given person depending on the

Fig. 15a: Extent of sympathy  $ES(x_\mu)$  depending on the long-term mean level  $x_\mu$  of well-being

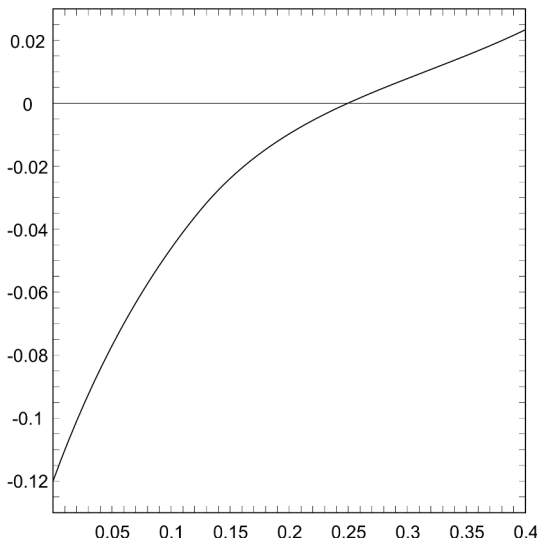


Fig. 15b: First derivation  $ES'(x_\mu)$  of the extent of sympathy depending on the long-term mean level  $x_\mu$  of well-being

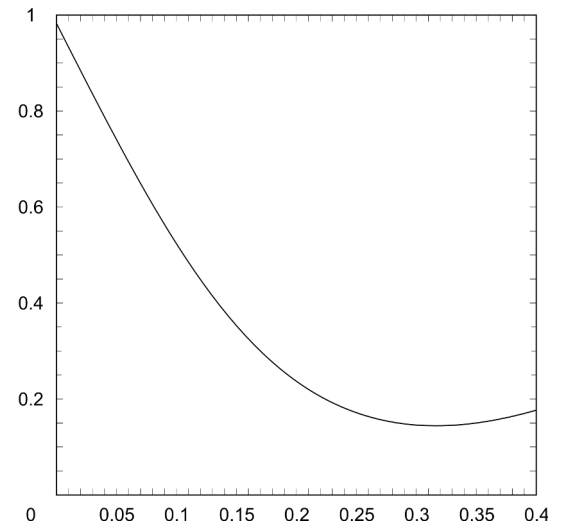


Fig. 16a: Normalized extent of sympathy  $ESN(u)$  depending on the long-term mean level  $u$  of well-being

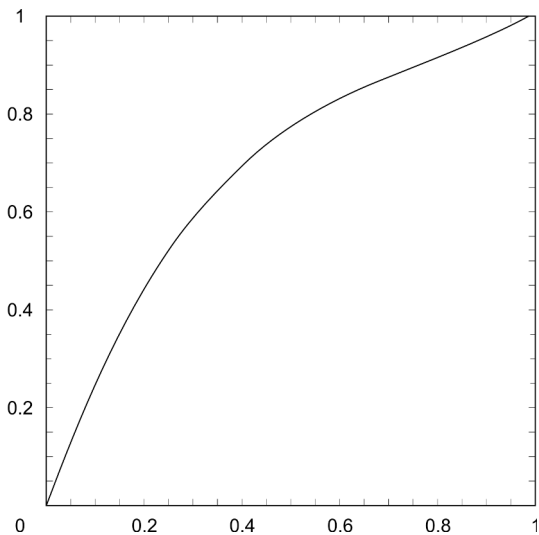
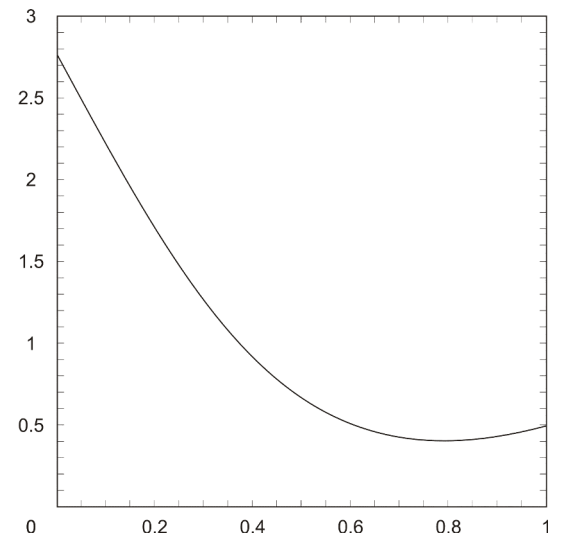


Fig. 16b: First derivation  $ESN'(u)$  of the normalized extent of sympathy depending on the long-term mean level  $u$  of well-being

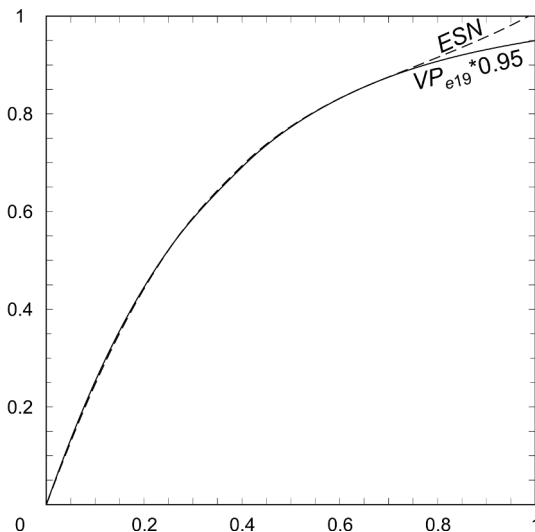


mean well-being  $x_\mu$  of this person. This operation can be repeated for all the long-term means  $x_\mu$  of well-being from the empirically expected range of such means, i.e. the interval from 0 to 0.4. (Please note that  $x_\mu$  this way has become a *variable*, representing different persons' general level of welfare.) The result is the function of the *extent of sympathy*  $ES(x_\mu)$  depending on the long-term mean level  $x_\mu$  of well-being. ( $ES(x_\mu) = \int_0^1 [PD(WBo, t=x) \cdot S(x)] dx$  (cf. figure 15).) Normalizing the long term mean levels of well-being  $x_\mu$  as well as the resulting extents of sympathy by a positive-linear transformation in the interval  $[0, 1]$  one gets the normalized function of the extent of sympathy:  $ESN(m)$ . Please consider figure 16. The  $x$ -axis represents the normalized mean-levels of well-being; and the  $y$ -axis represents the normalized expected extent of sympathy resulting from confrontations with persons having the respective mean-level of well-being. Because the mean level  $x_\mu$  of well-being in the framework adopted here is proportional to the integral of well-being over time, i.e. the total well-being of a person during her life, the normalized mean level  $m$  of well-

being with a hedonist theory of personal desirability can be equated with the total utility  $u$  of the respective person's life. This means that the function  $ESN$  in the end is running over  $u$  - as do the moral value functions considered above.

If somebody wants to value some social order from a purely sympathetic perspective he can assess the various mean levels of well-being, i.e. the levels of overall welfare, of the people living in this society, find out the appertaining extent of sympathy and, finally, sum up these extents of sympathy. This, of course, is the same procedure that a (hedonist) prioritarian has to undergo for assessing the prioritarian value of this social order. The only difference is that the prioritarian uses the prioritarian value function  $VP$  instead of the function of the extent of sympathy  $ESN$ . But comparing the latter function ( $ESN$ ) with the proposed exponential prioritarian welfare function  $util_{ex}$ , i.e.  $VP_{e19}$ , (cf. figure 17) one can easily see that the two functions for a big stretch are more or less identical. (For facilitating the comparison the prioritarian function has been compressed by the factor 0.95.)

Fig. 17: Comparison of the normalized extent of sympathy  $ESN(u)$  with  $util_{ex} VP_{e19} \cdot 0.95$  (exponential value function)



The function of the extent of sympathy  $ESN$  just presented is based on some rather provisional assumptions or measurements. But its general shape is rather stable with respect to changes of these assumptions. So remeasuring may change the exact function but the prioritarian shape will remain because it depends only on the stronger intensity of pity as compared to positive sympathy.

The main difference between the function of the extent of sympathy  $ESN$  and the particular prioritarian value function  $VP_{e19}$  is that the former at its far right end is slightly convex. This deviation may be explained as being due to a sympathetic ideal of perfection: We are delighted to an outstanding degree that there is somebody who is - compared to what humans can reach - nearly perfectly well-off. This ideal of perfection is only a small effect though; it leads to giving priority to the people best off only compared to people already very, very well off.





$$(4) \quad \sum_{i \leq (n+1)/2} (n+1-2i) \cdot u_m + \sum_{i > (n+1)/2} (n+1-2i) \cdot u_m + \\ \sum_{i \leq (n+1)/2} (n+1-2i) \cdot d_i + \sum_{i > (n+1)/2} (n+1-2i) \cdot (-d_i).$$

The first line of (4) is equal to 0 because the factors  $(n+1-2i)$  on the left side are always positive, whereas those on the right side are negative, and because for every factor  $(n+1-2i)$  on the left side there is a matching factor on the right such that the sum of both is equal to 0. This matching factor for  $i$  on the left side is  $n+1-i$  on the right side; so we get:  $(n+1-2i) + (n+1-2(n+1-i)) = 2n+2-2i-2n-2+2i = 0$ . (For e.g.  $n=10$  and  $i=1$  on the left side, the matching  $i$  on the right side is  $(10+1-1)=10$ . The left factor is  $(10+1-2 \cdot 1)=9$ ; the matching right factor is  $(10+1-2 \cdot 10)=-9$ .) There is one exception to this matching, though. If  $n$  is odd the middle  $i$  (for  $n=9$  this would be  $i=5$ ) will stand on the left side without having a corresponding  $i$  on the right side. But in this case the factor  $n+1-2i$  is equal to 0 anyway (in this case the middle  $i$  is equal to  $(n+1)/2$  so that the factor will become:  $n+1-2((n+1)/2) = n+1 - (n+1) = 0$ ). - With the first line of (4) being equal to 0, (4) reduces to: (4) =

$$(5) \quad \sum_{i \leq (n+1)/2} (n+1-2i) \cdot d_i + \sum_{i > (n+1)/2} (n+1-2i) \cdot (-d_i) =$$

(multiplying twice by -1 on the right side)

$$IE_{UJf} \quad \sum_{i \leq (n+1)/2} (n+1-2i) \cdot d_i + \sum_{i > (n+1)/2} (2i-n-1) \cdot d_i. \text{ Q.e.d.}$$

## References

- Anderson, Elizabeth S. (1999): What is the Point of Equality? In: *Ethics* 109. Pp. 287-337.
- Atkinson, Anthony B. (1970): On the Measurement of Inequality. In: *Journal of Economic Theory* 2. Pp. 244-263.
- Atkinson, A[nthony] B. (1975): *The Economics of Inequality*. Oxford: Clarendon Press. xi; 295 Pp.
- Atkinson, Anthony B.; Joseph E. Stiglitz (1980): *Lectures on Public Economics*. London [etc.]: McGraw-Hill. xvi; 619 pp.
- Boadway, R.; N. Bruce (1984): *Welfare Economics*. Cambridge, MA: Basil Blackwell.
- Broome, John (1989a): *Weighing Goods. Equality, Uncertainty and Time*. Oxford; Cambridge, Mass.: Blackwell. x; 255 pp.
- Broome, John (1989b): What's the good of equality? In: John D. Hey (ed.): *Current Issues in Microeconomics*. Basingstoke; London: Macmillan. Pp. 236-262.
- Broome, John (2002): Equality versus Priority. A Useful Distinction. Forthcoming in: C. Murray; Daniel Wikler (eds.): *Fairness and Goodness in Health*. World Health Organisation. (Ms. 2002. 7 pp.)
- Dalton, Hugh (1920): The Measurement of the Inequality of Incomes. In: *Economic Journal* 30. Pp. 348-361.
- Diamond, Peter A. (1967): Cardinal Welfare, Individual Ethics, and Interpersonal Comparison of Utility. Comment. In: *Journal of Political Economy* 75. Pp. 765-766.
- Drèze, Jean; Nicholas Stern (1987): The Theory of Cost-Benefit Analysis. In: Alan J. Auerback; Martin Feldstein (eds.): *Handbook of Public Economics*. Vol. II. Amsterdam [etc.]: North-Holland. Pp. 909-989.
- Fankhauser, Samuel; Richard S. J. Tol; David W. Pearce (1997): The Aggregation of Climate Change Damages. A Welfare Theoretic Approach. In: *Environmental and Resource Economics* 10. Pp. 249-266.
- Fleurbaey, Marc (2002): Equality versus Priority. How Relevant Is the Distinction? Forthcoming in: C. Murray; Daniel Wikler (eds.): *Fairness and Goodness in Health*. World Health Organisation. (Ms. 2001. 18 pp.)

- Frankfurt, Harry G. (1987): Equality as a Moral Ideal. In: *Ethics* 98. Pp. 21-43. - Reprinted in: Harry G. Frankfurt: *The Importance of What We Care About*. Cambridge: Cambridge U.P. 1988. Pp. 134-158. - Or in: Louis P. Pojman; Robert Westmoreland (eds.): *Equality. Selected Readings*. Oxford: Oxford U. P. 1997. Pp. 261-273. - Or in: Mane Hajdin (ed.): *The Notion of Equality*. Aldershot [etc.]: Ashgate 2001. Pp. 367-389.
- Frankfurt, Harry (1997): Equality and Respect. In: *Social Research* 64. Pp. 3-15. - Reprinted in: Harry G. Frankfurt: *Necessity, Volition, and Love*. Cambridge U.P. 1999. Pp. 146-154.
- Gaertner, Wulf (1992): Distributive Judgements. In: W[ulf] Gaertner; M[arlies] Klemisch-Ahlert: *Social Choice and Bargaining Perspectives on Distributive Justice*. Berlin [etc.]: Springer. Pp. 17-59.
- Gaertner, Wulf (1995): Distributive Justice. Theoretical Foundations and Empirical Findings. In: *Greek Economic Review* 17. Pp. 97-114.
- Hammond, P. J. (1979): Two Person Equity. In: *Econometrica* 47.
- Harsanyi, John C. (1955): Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility. In: *Journal of Political Economy* 63. Pp. 309-321. - Reprinted in: John C. Harsanyi: *Essays on Ethics, Social Behaviour, and Scientific Explanation*. Dordrecht; Boston: Reidel 1976. Pp. 6-23.
- Harsanyi, John C. (1975a): Can the Maximin Principle Serve as a Basis for Morality? A Critique of John Rawls's Theory. In: *American Political Science Review* 69. Pp. 594-606. - Reprinted in: John C. Harsanyi: *Essays on Ethics, Social Behaviour, and Scientific Explanation*. Dordrecht; Boston: Reidel 1976. Pp. 37-63.
- Harsanyi, John C. (1975b): Nonlinear Social Welfare Functions. Do Welfare Economists Have a Special Exemption from Bayesian Rationality? In: *Theory and Decision* 6. Pp. 311-332. - Reprinted in: John C. Harsanyi: *Essays on Ethics, Social Behavior, and Scientific Explanation*. Dordrecht; Boston: Reidel 1976. Pp. 64-85.
- Harsanyi, John C. (1977a): *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge: Cambridge U.P. x; 314 pp.
- Harsanyi, John C. (1977b): Morality and the Theory of Rational Behaviour. In: *Social Research* 44. - Reprinted in: Amartya Sen; Bernard Williams (eds.): *Utilitarianism and Beyond*. Cambridge: Cambridge U.P. 1982. Pp. 39-62.
- Hausman, Daniel M. (2001): Equality versus Priority. A Badly Misleading Distinction. (Ms. 7 pp.)
- Hurley, Susan L. (1989): *Natural Reasons. Personality and Polity*. New York; Oxford: Oxford U.P. xii; 462 pp.
- Klint Jensen, Karsten (1996): Measuring the Size of the Benefit and Its Moral Weight. In: Wlodek Rabinowicz (ed.): *Preference and Value. Preferentialism in Ethics*. (= *Studies in Philosophy* 1996:1). Lund: Dept. of Philosophy, Lund University. Pp. 114-145.
- Klint Jensen, Karsten (2002): What is the Difference between (Moderate) Egalitarianism and Prioritarianism? Ms. for the Conference of the European Society for Analytical Philosophy, June 2002 in Lund. 20 pp.
- Kutschera, Franz von (1982): *Grundlagen der Ethik*. Berlin; New York: de Gruyter. xiii; 371 pp.
- Lumer, Christoph (1997): *Utilex - Verteilungsgerechtigkeit auf Empathiebasis*. In: Peter Koller; Klaus Puhl (eds.): *Current Issues in Political Philosophy. Justice in Society and World Order*. Wien: Hölder-Pichler-Tempsky. Pp. 99-110.
- Lumer, Christoph (1999): Quellen der Moral. Plädoyer für einen prudentiellen Altruismus. In: *Conceptus* 32. Pp. 185-216.
- Lumer, Christoph (2000): *Rationaler Altruismus. Eine prudentielle Theorie der Rationalität und des Altruismus*. Osnabrück: Universitätsverlag Rasch. 652 pp.

- Lumer, Christoph (2002a): *The Greenhouse. A Welfare Assessment and Some Morals*. Lanham, Md.; New York; Oxford: University Press of America. ix; 117 pp.
- Lumer, Christoph (2002b): *Motive zu moralischem Handeln*. In: *Analyse & Kritik* 24. Pp. 163-188.
- McKerlie, Dennis (1984): *Egalitarianism*. In: *Dialogue* 23. Pp. 223-237.
- McKerlie, Dennis (1994): *Equality and Priority*. In: *Utilitas* 6. Pp. 25-42.
- McKerlie, Dennis (2002): *Understanding Egalitarianism*. Ms. for the congress of the European Society for Analytical Philosophy, June 2002 in Lund. 22 pp.
- Nagel, Thomas (1978): *Equality*. Tanner Lecture, Stanford University 1977. In: *Critica*. Pp. 3-27. - Reprinted in: Thomas Nagel: *Mortal Questions*. Cambridge: Cambridge U.P. 1979. Pp. 106-127.
- Nagel, Thomas (1991): *Equality and Partiality*. New York; Oxford: Oxford University Press. ix; 186 pp.
- Parfit, Derek (1995): *Equality or Priority? The Lindley Lecture*, University of Kansas, November 21, 1991. Lawrence, KS: University of Kansas. 4°; 42 pp. – Reprinted in: Matthew Clayton; Andrew Williams (eds.): *The Ideal of Equality*. St. Martin's Press 2000.
- Parfit, Derek (1997): *Equality and Priority*. In: *Ratio, New Series* 10. Pp. 202-221. - Reprinted in: Andrew Mason (ed.): *Ideals of Equality*. Oxford: Blackwell 1998. Pp. 1-20. - Or in: Mane Hajdin (ed.): *The Notion of Equality*. Aldershot [etc.]: Ashgate 2001. Pp. 427-446.
- Pigou, Arthur C. (1912): *Wealth and Welfare*. London: Macmillan.
- Rabinowicz, Wlodek (2001): *Prioritarianism and Uncertainty. On the Interpersonal Addition Theorem and the Priority View*. In: Egonsson; Josefson; Petersson; Ronnow-Rasmussen (eds.): *Exploring Practical Philosophy. From Action to Values*. Aldershot: Ashgate. Pp. 139-165.
- Rabinowicz, Wlodek (2002): *Prioritarianism for Prospects*. In: *Utilitas* 14. Pp. 2-21.
- Rawls, John B. (1958): *Justice as Fairness*. In: *Philosophical Review* 67. Pp. 164-194. - Reprinted in: Robert E. Goodin; Philip Pettit (eds.): *Contemporary Political Philosophy. An Anthology*. Oxford: Blackwell 1997. Pp. 187-202.
- Rawls, John [B.] (1971): *A Theory of Justice*. Cambridge, Mass.: The Belknap Press of Harvard U.P. XV; 607 pp. (Reprint: Oxford: Clarendon Press 1972. - London; Oxford; New York: Oxford U.P. 1973.)
- Rawls, John [B.] (1979): *Eine Theorie der Gerechtigkeit*. Übers. v. Hermann Vetter. Frankfurt, Main: Suhrkamp. 674 pp. (German 'translation' of Rawls 1971, but anticipating already the revised edition from 1999.)
- Rawls, John [B.] (1999): *A Theory of Justice. Revised Edition*. Oxford [etc.]: Oxford U.P. 1999. xxii; 538 pp.
- Raz, Joseph (1986): *The Morality of Freedom*. Oxford: Oxford U.P.
- Rescher, Nicholas (1966): *Distributive Justice. A Constructive Critique of the Utilitarian Theory of Distribution*. Indianapolis; New York; Kansas City: Bobbs-Merrill. xvi; 166 pp.
- Ross, Geoffrey (1974): *Utilities for Distributive Justice. The Meshing Problem and a Solution to it*. In: *Theory and Decision* 4. Pp. 239-258.
- Schoch, D[aniel] (1994): *An Axiomatic Basis for Distributional Equality in Utilitarianism*. In: *Erkenntnis* 40. Pp. 121-132.
- Schoch, Daniel (2004): *Fairness and Dynamic Consistency*. In: Roland Bluhm; Christian Nitz (eds.): *Ausgewählte Beiträge zu den Sektionen Philosophie der GAP.5, 5. Internationaler Kongress der Gesellschaft für Analytische Philosophie, Bielefeld, 22.-26. September 2003. (CD-ROM.)* Paderborn: Mentis. Pp. 754-760.
- Sen, Amartya K[umar] (1970): *Collective Choice and Social Welfare*. (San Francisco: Holden-Day.) Amsterdam; New York; Oxford: North-Holland 1984. xi; 225 pp.

- Sen, Amartya [Kumar] (1973): *On Economic Inequality*. Enlarged edition with a substantial annexe 'On Economic Inequality after a Quarter Century'. Oxford: Clarendon 1997. xiii; 260 pp.
- Sen, Amartya [K.] (1984): *Equality of What? (The Tanner Lecture on Human Values Delivered at Stanford University May 22, 1979.)* In: S. McMurrin (ed.): *The Tanner Lectures on Human Values*. Vol. I. Cambridge: Cambridge University Press. Pp. 195-220. - Reprinted in: Robert E. Goodin; Philip Pettit (eds.): *Contemporary Political Philosophy. An Anthology*. Oxford: Blackwell 1997. Pp. 476-486.
- Temkin, Larry S. (1993): *Inequality*. New York; Oxford: Oxford U.P. xiii; 352 pp.
- Temkin, Larry S. (2003): *Equality, Priority, or What?* In: *Economics and Philosophy* 19.
- Trapp, Rainer W. (1988): *"Nicht-klassischer" Utilitarismus. Eine Theorie der Gerechtigkeit*. Frankfurt: Klostermann. 715 pp.
- Trapp, Rainer W. (1990): *'Utilitarianism Incorporating Justice'. A Decentralised Model of Ethical Decision Making*. In: *Erkenntnis* 32. Pp. 341-381.
- Wagstaff, Adam (1991): *QALYS and the equity-efficiency trade-off*. In: *Journal of Health Economics* 10. Pp. 21-41.
- Wiggins, David (1987): *Claims of Need*. In: David Wiggins: *Needs, Values, Truth. Essays in the Philosophy of Value*. Oxford: Blackwell. Pp. 1-59.